TILA-S: Timing-Driven Incremental Layer Assignment Avoiding Slew Violations

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Abstract—As VLSI technology scales to deep submicron and beyond, interconnect delay greatly limits the circuit performance. The traditional 2D global routing and subsequent net by net assignment of available empty tracks on various layers lacks a global view for timing optimization. To overcome the limitation, this paper presents a timing driven incremental layer assignment tool, TILA-S, to realign layers among routing segments of critical nets and non-critical nets. Lagrangian relaxation techniques are proposed to iteratively provide consistent layer/via assignments. Modeling via min-cost flow for layer shuffling avoids using integer programming and yet guarantees integer solutions via uni-modular property of the inherent model. In addition, multiprocessing of $K \times K$ partitions of the whole chip provides runtime speed up. Furthermore, a slew targeted optimization is presented to reduce the number of violations incrementally through iteration-based Lagrangian relaxation, followed by a post greedy algorithm to fix local violations. Certain parameters introduced in the models provide trade-off between timing optimization and via count. Experimental results in both ISPD 2008 and industry benchmark suites demonstrate the effectiveness of the proposed incremental algorithms.

Index Terms—Global routing, layer assignment, timing, min-cost network flow.

I. INTRODUCTION

As VLSI technology scales to deep submicron and beyond, interconnect delay plays a determining role in timing [1]. Therefore, interconnect synthesis, including buffer insertion / sizing and timing-driven routing, becomes a critical problem for achieving timing closure [2]. Global routing is an integral part of a timing convergence flow to determine the topologies and layers of nets, which greatly affect the circuit performance [3]–[9]. In emerging technology nodes, back-end-of-line (BEOL) metal stack offers heterogeneous routing resources, i.e., dense metal at the lower layers and wider wires at the upper layers. Fig. 1 gives one example of cross section of IC interconnection stack in advanced technology nodes [10], where wires and vias on top metal layers are much wider and much less resistive than those on lower metals. Besides, the normalized pitches of different metal layers from [11] are also listed. Advanced routing algorithms should not only be able to achieve routability, but also intelligently assign layers to overcome interconnect timing issues.

Layer assignment is an important step in global routing to assign each net segment to a metal layer. It is commonly generated during or after the wire synthesis to meet tight frequency targets, and to reduce interconnect delay on timing critical paths [12]. In layer assignment, wires on thick metals are much wider and thus, less resistive than those on thin metals. If timing critical nets are assigned to lower layers, it will make timing worse due to narrower wire width/spacing. Although top metal layers are less resistive than those in lower (thin) metals, it is impossible to assign all wires to top layers. That is, layer assignment should satisfy the capacity constraints on metal layers. If an excessive number of wires are assigned to a particular layer, it will aggravate congestion and crosstalk. Meanwhile, the delay due to vias cannot be ignored in emerging technology nodes [1]. In addition, during timing closure slew violations could affect the utilization of buffering resources [13]. Thus to guarantee signal integrity and reduce buffering resources, slew violations need to be avoided during layer assignment.

Recently, layer assignment has been considered in two design stages, i.e., buffered tree planning and 3D global routing. Some studies consider layer assignment during buffer routing trees design [12], [14], [15]. Li et al. [12] proposed a set of heuristics for simultaneous buffer insertion and layer assignment. Hu et al. [14], [15] proved that, even if buffer positions are determined, the layer assignment with timing constraints is $NP$-complete. During 3D global routing, layer assignment is a popular technique for via minimization. Cho et al. [3] proposed an integer linear programming (ILP) based method to solve the layer assignment problem. Since via minimization is the major objective, all wires tend to be assigned onto the lower layers. [16], [17] applied dynamic programming to solve optimal layer assignment for a single net. To overcome the impact of net ordering, different heuristics or negotiation techniques were proposed in [18], [19]. Ao et al. [19] considered the delay in layer assignment, but since via capacity was not considered, more segments can be illegally pushed onto higher routing layers. A min-cost flow based refinement was developed in [20] to further reduce the number of vias. Furthermore, Lee et al. [21] proposed an enhanced global router with layer assignment refinement to reduce possible violations through min-cost max-flow network. This framework works at one edge each time in a sequential order. For slew optimization, repeaters/buffers insertions are widely adopted to fix the potential slew violations [12], [13], [22]. Zhang et al. [23] utilized an ILP approach to reconstruct the over-the-block steiner tree structure to improve slew.

Existing layer assignment studies suffer from one or more of the following limitations: (1) Most works only target at via number minimization, but no timing issues are considered. Since timing requirements within a single net are usually different for different sinks, assigning
all segments of a set of nets on higher metal layers is not the best use of critical metal layer resources. That is, intelligent layer assignment should not blindly assign all segments of a net to a set (a pair, for example) of higher metal layers. It should be aware of capacitive loading of individual segments within a net to achieve better timing with the limited available higher metal layer resources. (2) In emerging technology nodes, the via delays contribute a non-negligible part of total interconnect delay. But the delay impact derived from vias is usually ignored in previous layer assignment works. (3) During post routing stage, slew violations may cause more buffering resources. There are limited works to avoid slew violations globally during layer assignment stage. (4) The net-by-net strategy may lead to local optimality, i.e., for some nets the timings are over-optimized, while some other nets may have no enough resources in high layers. Meanwhile, considering one edge at each time may lose potential optimality because the edge ordering could also affect the subsequent solutions.

To close on timing for critical nets that need to go long distances, layer assignment needs to be controlled by multi-net global optimization. For example, Fig. 2 compares the delay distributions of benchmark ‘adaptec2’ by conventional layer assignment solver [18] and our incremental timing-driven solution, while Fig. 3 compares the slew distribution results. We can see that, since conventional layer assignment only targets at via minimization, the maximum delay and the maximum slew can be very large. Since our timing-driven planner is with global view, the maximum delay can be much better, i.e., the normalized maximum delay can be reduced from $144 \times 10^5$ to $23 \times 10^5$. Meanwhile, the slew violations can also be reduced significantly. The maximum slew decreases from $12.74 \times 10^5$ to $2.16 \times 10^5$. Here we do not take buffered information into accounts for this benchmark.

For very large high-performance circuits, either long computation times have to be accepted or routing quality must be compromised. Therefore, an incremental layer assignment to iteratively improve routing quality is a must. In this paper, we propose an incremental layer assignment framework targeting at timing optimization. Incremental optimizations or designs are very important in physical design and CAD field to achieve good timing closure [24]. Fast incremental improvements are developed in different timing optimization stages, such as incremental clock scheduling [25], [26], incremental buffer insertion [27], and incremental clock tree synthesis [28]. To further improve timing, incremental placement is also a very typical solution [29], [30]. Besides, there are several incremental routing studies (e.g. [31]) to introduce cheap and incremental topological reconstruction.

To the best of our knowledge, this work is the first incremental layer assignment work integrating via delay and solving all the nets simultaneously. A multilayer global router can either route all nets directly on multilayer solution space [4], [5] or 2D routing followed by post-stage layer assignment [6]–[9]. Note that as an incremental layer assignment solution, our tool can smoothly work with either type of global router. Our contributions are highlighted as follows.

- A mathematical formulation gives the layer assignment solutions with optimal total wire delays and via delays.
- A Lagrangian relaxation based optimization iteratively improves the layer assignment solution.
- Lagrangian relaxation subproblem (LRS) is solved via min-cost flow model that guarantees integer solutions due to inherent unimodular property, thus, avoiding runtime extensive methods such as ILP.
- An iterative Lagrangian relaxation based slew optimization strategy is proposed to reduce the violations globally.
- A post slew optimization algorithm searches potential usable layers for fixing local violations.
- Multiprocessing of $K \times K$ partitions of the whole chip provides runtime speed up.
- Both ISPD 2008 and industry benchmarks demonstrate the effectiveness of our framework.

The remainder of this paper is organized as follows. Section II provides some preliminaries and the problem formulation. Section III gives mathematical formulation, and also proposes sequence of
multi-threaded min-cost flow algorithm to achieve further speed-up. In addition, mitigating slew violations is discussed in this Section. Section IV reports experimental results, followed by conclusion in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section we introduce the graph model and the timing model applied in this paper. Then the problem formulation of timing-driven incremental layer assignment is provided.

A. Graph Model

Similar to the 3D global routing problem, layer assignment problem can be modeled on a 3D grid graph, where each vertex represents a rectangular region of the chip, so called a global routing cell (G-Cell), while each edge represents the boundary between two vertices. In the presence of multiple layers, the edges in the z-direction represent vias connecting different layers. Fig. 4(a) shows a grid graph for routing a circuit in multi-metal layer manufacturing process. Each metal layer is dedicated to either horizontal or vertical wires. The corresponding 3D grid graph is shown in Fig. 4(b).

To model the capacity constraint, for each x/y-direction edge, we denote its maximum routing capacity as \(c_{e_{x/y}}\). Besides, the via capacity of each vertex, denoted by \(c_{v}\), is computed as in [32]. In brief, via capacity refers to the available space for vias passing through the cell, and is determined by the available routing capacity of those two x/y-direction edges connected with the vertex. If there is no routing space for those two edges, no vias are allowed to be inserted in this cell. Thus, this via capacity model helps to keep adequate routing space for vias through layers, and places the limits of wires on higher metal layers, which may result in wire delay degradation.

B. Delay Model

We are given a global routing of nets, where each net is a tree topology with one source and multiple sinks. Based on the topology, for each net we have a set of segments \(S\). Here we give an example of net model in Fig. 5, where each net contains two segments. To evaluate the timing of each net, we adopt Elmore delay model, which is widely used during interconnect synthesis in physical design. The delay of a segment \(s_i\) on a layer \(l\), denoted by \(d_e(i,l)\), is computed as follows:

\[
d_e(i,l) = R_e(l) \cdot \left(\frac{C(l)}{2} + C_{\text{down}}(s_i)\right),
\]

where \(R_e(l)\) and \(C(l)\) refer to the edge resistance on layer \(l\), and edge capacitance on layer \(l\), respectively. \(C_{\text{down}}(s_i)\) refers to the downstream capacitance of \(s_i\). Note that the downstream capacitance of \(s_i\) is determined by the assigned layers of its all downstream segments. To calculate the downstream capacitance for each \(s_i\), we should traverse the net tree from sinks to source in a bottom-up manner. Therefore, the downstream capacitance of the source segment, i.e. the segment connected with the driver pin, should be calculated after all the other segments have obtained their downstream capacitances.

For a via \(v_m\) connecting segments between layers \(l\) and \(l+1\), its delay can be calculated as follows:

\[
d_e(v_m,l) = R_e(l) \cdot C_{\text{down}}(v_m).
\]

Here \(R_e(l)\) is the resistance of via between layers \(l\) and \(l+1\), and \(C_{\text{down}}(v_m)\) is the downstream capacitance of the upstream segment connected to via \(v_m\). If the downstream capacitance of a via is equal to zero, then we assume the via delay is negligible.

In addition, buffer positions can be considered in our delay model. That is, for one segment \(s_i\), if there is one buffer at its end point, its downstream capacitance \(C_{\text{down}}(s_i)\) should be equal to the buffer input capacitance. As shown in Fig. 5, \(C_{\text{down}}(s_2)\) is equal to the input capacitance of the buffer. Because buffers are fabricated in silicon and have pins connected with a specified metal layer, integration with buffers in our assumption would affect the downstream capacitance for the corresponding pin. Meanwhile, integration with buffers would also introduce buffer intrinsic delay and driving delay for each driving net. The intrinsic delay is dependent on the driving buffer, while the driving delay is in proportion to the downstream capacitance. Because capacitances of different layers vary less than resistances, we do not include the buffer driving delay in our work. Therefore, through updating the downstream capacitances and including buffer intrinsic delay, our framework can handle timing optimization for both pre-buffered and post-buffered designs.

C. Slew Model

Besides delay, our framework also considers slew computation to reduce the potential slew violations. Since each routing net is a tree topology in essence, we traverse the tree in a breath-first manner from the driver to each sink and calculate the slew for each pin. For each segment, the input slew is represented by its upstream pin slew, and the output slew by its downstream pin slew. To calculate the output slew, we adopt PERI model, which has been shown to provide less than 1% error [33]. The calculation is given in Eq. (3), where \(Slw(p_a(s_i))\) and \(Slw(p_d(s_i))\) are the input and output slew of \(s_i\), respectively, while \(Slw_{\text{step}}(s_i)\) is the step slew.

\[
Slw(p_d(s_i)) = \sqrt{Slw(p_a(s_i))^2 + Slw_{\text{step}}(s_i)^2}.
\]

Based on PERI model, the segment output slew depends on both its input slew and step slew. The input slew is also the output slew of the upstream segment, so it can be obtained iteratively through Eq. (3). Regarding the step slew, we calculate it through the combination of PERI model and Bakoglu’s metric. It is proved to have error within 4% [33]. The calculation is shown in Eq. (4), where \(l(s_i)\) is the layer on which \(s_i\) is assigned, and \(d_e(i,l)\) is Elmore delay of segment \(s_i\) on layer \(l\).

\[
Slw_{\text{step}}(s_i) = Slw_{\text{step}}(i,l(s_i)) = \ln 9 \cdot d_e(i,l(s_i)).
\]

With the calculated step slew, we can obtain the output slew for each segment. To see the impact of layer assignment, the output slew can be represented as a function of its input slew and the layer to be assigned.

\[
Slw_e(i,l(s_i)) = \sqrt{Slw(p_a(s_i))^2 + \left(\ln 9 \cdot d_e(i,l(s_i))\right)^2}.
\]

Besides, via slew should also be considered during slew calculation and computed in a similar way as segment slew. Eq. (6) gives the slew
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Fig. 6: An example of timing driven layer assignment. In initial layer assignment net \( n_3 \) is timing critical net. Through resource releasing from nets \( n_1 \) and \( n_2 \), the total timing gets improvement.

For via \( v_m \) from layer \( l \) to layer \( l+1 \).

\[
SLw_v(v_m, l+1) = \sqrt{SLw(p_{v_m})^2 + (ln9 \cdot d_v(v_m, l))^2}.
\]

In contrary to downstream capacitance calculation in a bottom-up manner, here we start from the segment connected with the net driver. Then each segment and its connected via are traversed in a breadth-first manner until every sink is reached. With this approach, we obtain the output slew for each net sink sequentially. If the sink slew exceeds a specified slew constraint, we assume there is a slew violation.

D. Problem Formulation

Based on the grid model and timing model discussed in the preceding section, we define the timing-driven incremental layer assignment (TILA) problem as follows:

**Problem 1 (TILA).** Given a global routing grid, a set of critical net segments and layer via capacity information, timing-driven incremental layer assignment assigns each segment passing through an edge to a layer, so that layer assignment costs (weighted sum of segment delays, via delays, and slew violations) can be minimized, while the capacity constraints of each edge on each layer are satisfied.

It shall be noted that in this work we only consider layer assignment for timing optimization, while other techniques such as buffering are not discussed. One instance of TILA problem with three nets is demonstrated in Fig. 6, where nets \( n_1 \) and \( n_2 \) are non-critical nets, while net \( n_3 \) is timing critical net. In the initial layer assignment, net \( n_3 \) is assigned lower layers. Since the routing resources are utilized by nets \( n_1 \) and \( n_2 \), \( n_3 \) cannot be shuffled into higher layers to improve timing. Through a global layer reassignment, we are able to achieve a better timing assignment solution, where both \( n_1 \) and \( n_2 \) release high layer resources to \( n_3 \).

Nacerio et al. proved that even if no timing is considered, the decision version of layer assignment for via minimization is \( \mathcal{NP} \)-complete [34]. Thus the decision version of TILA problem is \( \mathcal{NP} \)-complete as well.

III. TILA Algorithms

In this section, we introduce our framework to solve the TILA problem. First a mathematical formulation targeting delay optimization will be given. Then a Lagrangian relaxation based optimization methodology is proposed to solve this problem. After the delay optimization, a Lagrangian relaxation based slew optimization is presented, followed by a post optimization stage. For convenience, some notations used in this section are listed in TABLE 1.

### A. Mathematical Formulation

The starting mathematical formulation of TILA problem is shown in Formula (7). In the objective function, the first term is to calculate the cost from segments, while the second term is to calculate the cost from vias. Here \( d_v(i, j) \) is calculated through Eq. (1), and \( d_c(i, p, k) \) is derived from Eq. (2).

\[
\begin{align*}
\min & \quad \sum_{i \in S} \sum_{j=1}^{L} d_e(i, j) \cdot a_{ij} + \\
& \sum_{(i,j) \in E_x} \sum_{k=1}^{\max(l(i,j))} \sum_{q=1}^{\max(l(i,j))} d_v(i, p, k) \cdot a_{ij} \cdot a_{pq}, \\
\text{s.t.} & \quad \sum_{j} a_{ij} = 1, \quad \forall i \in [1, S], \\
& \quad \sum_{s_i \in S_e(i)} a_{ij} \leq c_e(i,j), \quad \forall e \in E, \forall j \in [1, L], \\
& \quad \sum_{(i,j) \in E_x(g)} a_{ij} \cdot a_{pq} \leq c_g(k), \\
& \quad a_{ij} \text{ is binary }.
\end{align*}
\]

Constraint (7b) is to ensure that each segment of nets would be assigned to one and only one layer. Each edge \( e \in E \) is associated with one capacity \( c_e(i,j) \), and constraint (7c) is for the edge capacity of each layer. Constraint (7d) is for the via capacity in each layer, which restricts the available via capacity for each layer at certain grid position.

First, we show that if each \( C_{down}(s_i) \) is constant, the TILA problem can be formulated as an integer linear programming (ILP), then a mature ILP solver is possible to be applied. Here \( C_{down}(s_i) \) is downside capacitance of segment \( s_i \). We can use a boolean variable \( \gamma_{ij,pq} \) to replace each non-linear term \( a_{ij} \cdot a_{pq} \). Then Formula (7) can be transferred into ILP through introducing the following artificial constraints:

\[
\begin{align*}
\{ a_{ij} + a_{pq} \leq & \gamma_{ij,pq} + 1, \\
& a_{ij} \geq \gamma_{ij,pq}, \quad a_{pq} \geq \gamma_{ij,pq} \}.
\end{align*}
\]

Due to the computational complexity, ILP formulation suffers from serious runtime overhead, especially for those practical routing test cases. A popular speedup technique is to relax the ILP into linear programming (LP) by removing the constraint (7e). It is obvious that the LP solution provides a lower bound to the original ILP formulation. We observe that the LP solution would be like this: each \( a_{ij} \) is assigned to 0.5 and each \( \gamma_{ij,pq} \) is 0. By this way, all the constraints are satisfied, and the objective function is minimized, however all these 0.5 values to
Algorithm 1 TILA

Require: Initial layer assignment solution;

Require: Critical net ratio α;

1: Select all segments based on α; \(\triangleright\) Section III-D
2: Initialize \(C_{down}(s_i)\) for each segment \(s_i\);
3: Initialize LM;
4: while not converged do
5: \(\triangleright\) Section III-C
6: Update \(C_{down}(s_i)\) for all \(s_i\);
7: Update LM;
8: end while

Algorithm 1 gives a high level description of our Lagrangian relaxation based framework to the TILA problem. The inputs are an initial layer assignment solution and a critical net ratio value \(\alpha\). Based on the \(\alpha\) value we select some critical nets and non-critical nets (line 1). All the segments belonging to these (selected critical and non-critical) nets are reassigned layers by our incremental framework. Please refer to Section III-D for more details of our critical and non-critical net selection. Based on the initial layer assignment solution, we initialize all the \(C_{down}(s_i)\) for each selected segment \(s_i\) (line 2). The LMs are also initialized in line 3. In our implementation, the initial values of all LMs are set to 0.000. Our framework iteratively solves a set of Lagrangian relaxation subproblems (LRS), with fixed LM values (lines 4–8). In solving LRS, we minimize the objective function in Eq. (9) based on the current set of LMs. The details of solving LRS are discussed in Section III-C. After solving each LRS, we re-calculate the downstream capacitances of all the segments \(C_{down}(s_i)\) based on Eq. (1) (line 6). We use a subgradient-based algorithm [36] to update the LMs to maximize LDP (line 7). In more details, the LM in the current iteration is dependent on the LM from the last iteration \(\lambda_{i,j,p,q}^{\prime}\) the step length \(\theta_{i,j,p,q}\), and the available resources.

\[
\lambda_{i,j,p,q} = \lambda_{i,j,p,q}^{\prime} + \theta_{i,j,p,q} \cdot (a_{ij} \cdot a_{pq} - c_{g}).
\]

The available via resources can be obtained directly by updating the current via capacity as in [32]. To decide the step length, we adopt the classic calculation as follows:

\[
\theta_{i,j,p,q} = \frac{\phi \cdot [UB - L(\lambda_{i,j,p,q})]}{\|a_{ij} \cdot a_{pq} - c_{g}\|^2}.
\]

Based on Eq. (11), \(UB\) refers to the upper bound of the total costs of via and segments connecting to \(v\) and \(L(\lambda_{i,j,p,q})\) refers to the current total costs. \(\phi\) is the scaling factor traditionally from 2 to 0, and here we choose it as 1 for convenience. Through this updating procedure, LMs help to fix the potential via violations. In our implementation, the iteration in line 4 will end if one of the following two conditions is satisfied: either the iteration number is larger than 20; or both the wire delay improvement and the via delay improvement are less than a pre-specified fraction.

C. Solving Lagrangian Subproblem (LRS)

Through removing the constant items and reorganizing objective function of Formula (9), we re-write LRS into Formula (12).

\[
\min \sum_{i=1}^{S} \sum_{j=1}^{L} c(i,j) \cdot a_{i,j} + \sum_{(i,p) \in E_x} \sum_{j=1}^{L} \sum_{q=1}^{\max} d_s(i,p,k) \cdot a_{ij} \cdot a_{pq},
\]

\[
\text{s.t.} \quad (7b) - (7c), (7e),
\]

Theorem 1. For a set of fixed \(\lambda_{i,j,p,q}\), LRS is \(NP\)-hard.

Due to space limit, the detailed proof is omitted. Because of nonlinear term \(a_{ij} \cdot a_{pq}\), the proof can be through a reduction from quadratic assignment problem [37]. In addition, unless \(P = NP\), the quadratic assignment problem cannot be approximated in polynomial time within some finite approximation ratio [38]. Inspired by MacCormick Envelops, we prefer to linearize the term \(a_{ij} \cdot a_{pq}\).

\[
c(i,j,p,q) \cdot a_{ij} \cdot a_{pq} \approx c(i,j,p,q) \cdot (a_{ij}^t \cdot a_{pq} + a_{ij} \cdot a_{pq}^t),
\]

where \(a_{ij}^t\) and \(a_{pq}^t\) are the value of \(a_{ij}\) and \(a_{pq}\) in previous iteration, and \(a_{ij}^t\) is the value of \(a_{ij}\) in previous iteration. This linearization is based on the segment assignment of the last iteration. Since LRS is solved iteratively through updating LMs, this approximation is acceptable. Taking \(a_{18} \cdot a_{29}\) as an instance, where \(a_{18}\) and \(a_{29}\) are 1, we can obtain that segments \(s_1\) and \(s_2\) are assigned on layers 8 and 9 in the previous iteration, respectively. This means that segments \(s_1\) and \(s_2\) should belong to critical nets because they have been assigned on high metal layers by our framework. Thus, in later iterations, when considering the assignment of segment \(s_1\), we assume that segment \(s_2\) is assigned on layer 9, and vice versa, according to Eq. (13). In this manner, segments \(s_1\) and \(s_2\) are as probable to be assigned on high metal layers as before. Since each critical segment has a tendency to be assigned on high metal layers, the problem converges after several iterations.

Through the linearization technique in Eq. (13), the objective function in Formula (12) is a weighted sum of all the \(a_{ij}\). We will show that the
linearized LRS can be solved through a min-cost network flow model. The basic idea is that the weighted sum of all the \( a_{ij} \) can be viewed as several assignments from segments to layers, while the weight of each \( a_{ij} \) is the cost to assign segment \( i \) to layer \( j \). Constraints (7b) and (7c) can be integrated into the flow model through specified edge capacity. Constraint (7e) is satisfied due to the inherent uni-modal property of min-cost network flow [36].

An example of such min-cost flow model is illustrated in Fig. 7. Given four different segments \( s_1, s_2, s_3, s_4 \) and several edges, we build up a directed graph \( G = (V, E) \) to represent the layer assignment relationships. The vertex set \( V \) includes four parts: start vertex \( s \), segment vertices \( V_s \), layer vertices \( V_L \), and end vertex \( t \). Here both start and end vertices are pseudo vertices. Segment vertices \( V_s \) represent a collection of segments to be assigned, where the collection size is equal to the number of segments. Similarly, a layer vertex in \( V_L \) represents a layer on which a segment can be reassigned. The edge set \( E \) is composed of three sets of edges: \{ \( s \rightarrow V_s \} \), \{ \( V_s \rightarrow V_L \} \), and \{ \( V_L \rightarrow t \} \). Notably, here the edge set \( E \) represents the edges in the network flow, while the layer vertices represent the layers of edges in the global routing grid model. We define all the edge costs as follows: the cost of one edge from \( V_s \) to \( V_L \) is the cost of assigning the segment to corresponding layer; the costs of all other edges are set to 0. For segments whose directions are not compatible with certain layers, no edge exists between those segment and layer vertices. We define all the edge capacities as follows: the capacity of one edge from \( V_L \) to node \( t \) is the capacity of the corresponding edge in the routing grid model; while the capacities of all other edges are set to 1. Then edge capacity constraint can be satisfied by the capacity of edge from \( V_L \) to node \( t \), and the capacity from node \( s \) to \( V_s \) guarantees that one segment can just be assigned on one layer. As shown in Fig. 7, segment \( s_1 \) can be assigned on either layer 6 or layer 8 of edge 1; similarly, segment \( s_2 \) can also be assigned on two layers of edge 2. The numbers shown in \( V_s \) vertices indicate the specified layer of this edge and the corresponding edge index, respectively. The corresponding grid model is given in Fig. 5, where we can see that segment \( s_1 \) shares the same routing edge with \( s_3 \), therefore \( s_1 \) competes the routing resource with \( s_3 \). Meanwhile, segment \( s_4 \) has a different routing direction with the other three segments so it has to be assigned on other layers for vertical routing. When the number of segments to be assigned on one edge exceeds the edge routing capacity, our framework will assign the segments in order to minimize the assigning costs. In this example, we assume that each segment passes through one edge with its length equal to the grid size, as shown in Fig. 5. For a segment passing through multiple edges, we prefer to split it into a set of sub-segments, and each sub-segment has the same length as the grid size. We construct the flow graph where each sub-segment has its own assigning cost, and the number of sub-segments to be assigned on one layer is also constrained by the layer node.

![Fig. 7: An example of min-cost flow model.](image)

![Fig. 8: Our parallel scheme to support multi-threading computing on \( K \times K \) partitions. (Here \( K = 4 \).) (a) Parallel pattern 1; (b) Parallel pattern 2.](image)

### D. Critical & Non-Critical Net Selection

Given an input ratio value \( \alpha \), our framework would automatically identify \( \alpha \)% of the total nets as critical nets, while other \( \alpha \)% of the total nets as non-critical nets. Both the selected critical nets and the selected non-critical nets would be reassigned layers. The motivation of critical net selection is to reassign their layers to improve timing, while the motivation of non-critical net selection is to release some high layer resources to the critical nets. By this way, our incremental layer assignment flow is able to overcome the limitation of any net order in original layer assignment. In our implementation, the default value of \( \alpha \) is set to 1, which means 1% of nets would be identified as critical nets, while the other 1% of nets are selected as non-critical nets.

To identify all the critical nets can be trivial: first all the net timing costs in original layer assignment are calculated based on our delay model as in Section II, and then the \( \alpha \)% of worst delays are selected. Yet, non-critical net selection is not so straightforward, as randomly selecting \( \alpha \)% of best timing nets may not be beneficial to improve critical net timing. Therefore, we prefer to select those nets with best timing sharing more routing resources with the critical nets while these nets are assigned on high metal layers. Otherwise, releasing the non-critical nets on lower layers have no benefits for final timing results. In our implementation, we check the \( 2 \cdot \alpha \)% best timing nets to associate each net with a score to indicate their overlapping resources with critical nets. Meanwhile, if there is an overlap with critical nets, the assigned layer of this short net should be higher than the lowest layer of these critical nets. Otherwise, it is not regarded as an effective overlap. Then we select half of them with the best scores as non-critical nets.

### E. Parallel Scheme

Our framework supports parallel scheme by dividing the global routing graph into \( K \times K \) parts. An example of such division is illustrated in Fig. 8, where \( K = 4 \). The timing-driven incremental layer assignment is solved in each partition separately. During partitioning, each segment is ensured to be solved in one and only one partition. To achieve this, for segments crossing boundaries between different partitions, they are assigned in the same partition as its geometric center. If its geometric center is exactly on the boundary, we assume this segment belongs to the partition in its left/bottom side. The reason of such division is twofold. Firstly, our Lagrangian relaxation based optimization is to solve a set of min-cost flow models, as discussed in Section III-B and Section III-C. The runtime complexity to solve a single flow model is \( \mathcal{O}(|V| \cdot |E|) \), where \( |V| \) and \( |E| \) are the vertex number and the edge number of the graph. Dividing the whole problem into a set of sub-problems can achieve significant speed-up. In addition, multi-threading is applied to provide further speed-up. For instance, in Fig. 8(a) four threads are used to solve different regions simultaneously. Secondly, inspired by the Gauss-Seidel method [39], when one thread is solving flow model in one partition, the most recently updated results by peer
threads are taken into account, even if the updating occurs in the current iteration. Besides the above example, we also propose a more general type of parallel pattern suitable for any $K \times K$ partition, as illustrated in Fig. 8(b). In this example, neighboring threads start in inverse directions and avoid operating on neighboring partitions simultaneously as much as possible. After solving different partitions, we synchronize the newly updated layer assignment results to eliminate the potential conflicts. This second pattern is more suitable for multi-processing considering its synchronization mechanism.

F. Iterative Slew Optimization

During timing closure, slew violations are important performance metrics that may cause a huge demand for buffering resources. Thus, we should also focus on reducing the number of slew violations besides delay optimization. Fig. 9 depicts the overall algorithm flow, which mainly consists of two stages: delay optimization and slew optimization. The details of delay optimization are already introduced from Section III-B to Section III-E. As discussed in Section II-C, segment step slew is in proportion to its delay. With the constant segment input slew, the higher layer this segment is assigned, the fewer output slew can be obtained. Therefore, delay optimization is deemed to mitigate slew violations. Nevertheless, segment delay optimization mainly considers the layer assignments of its downstream segments due to the existence of downstream capacitance, but neglects its upstream segments. Since layer assignments of the upstream segments affect the segment input slew, the upstream segments should also be taken into accounts.

An example is given in Fig. 10. Here we assume that both net $n_1$ and net $n_2$ are critical while there is only one available routing capacity for each edge, so segments $s_1$ and $s_2$ should compete for the higher layer resource. Regarding delay optimization, segment $s_2$ is possible to be assigned on a higher layer because it owes a larger downstream capacitance with a closer distance to its driver; while in fact, segment $s_1$ should be placed on a higher layer because it is on a longer path which may introduce slew violations. Through slew optimization flow as shown in Fig. 9, segment $s_1$ will be assigned a higher priority on a higher layer. The details of the algorithm flow will be given later. The main reason is that slew optimization considers the impact of both upstream segments and downstream segments. In this manner, slew optimization has a different impact on assignment of critical nets in comparison to delay optimization. If we consider both optimizations simultaneously, they may affect each other to degrade the final performance. The detailed reasons are two-fold: First, critical nets can be selected in a different way during delay and slew optimization. In the stage of slew improvement, these nets exceeding slew constraints are to be selected as critical nets to fix their violations; however in the first stage we mark these nets with higher total delays as critical nets. This may induce potential discrepancies for nets to be optimized. Secondly, delay improvement targets at total delay reduction considering via overflows, while slew improvement targets at slew violations reduction. Due to different optimal objectives, assigning costs for both delay and slew optimization may lead to a trade-off based on their weights. Considering the assigning differences of $s_1$ and $s_2$ in Fig. 10, possible oscillation may be introduced by setting different weights to delay and slew optimization. Therefore, due to the differences of selected nets and optimal objectives, we prefer to target delay and slew separately in an explicit manner, and reduce slew violations globally as a second stage after delay optimization.

Fig. 9 also outlines the slew optimization flow, whose input is the assignment result after delay optimization. The slew optimization consists of two steps: iterative slew optimization and post greedy optimization. This section focuses on the first step to reduce slew violations based on flow model, while Section III-G provides the details of post slew optimization. Some notations used in slew optimization are listed in TABLE II.

In the iterative optimization, similar with delay optimization flow, the same ratio of critical and non-critical nets are selected based on their slew. To calculate the net criticality, we divide the net into a set of paths, and calculate the sink slew of each path. If the sink slew exceeds the given slew constraint, this path is defined as a critical path, i.e. $P_{critical}$, and the exceptional slew is counted as critical value. Meanwhile, segment input slews are initialized based on the input result because each segment should be reassigned simultaneously. Then we reassign these nets through iteration-based Lagrangian relaxation optimization. When the number of slew violations converges to a certain ratio, the iteration-based optimization stops.

Now we go over the details about how to solve the problem through min-cost flow model. First all the segments on critical paths are considered because their layer assignments affect the path sink slew. During slew optimization, we lower the slew constraint by 5% in order to leave enough slew slacks. Eq. (14) gives the slew constraint:

$$\delta Slew(p_s(s_i)) \leq 0.95 \cdot Slew_c, \quad i \in P_{critical},$$  \hspace{1cm} (14)

where $Slew(p_s(s_i))$ is the segment output slew, and $Slew_c$ is the slew constraint. To solve this problem, we relax Eq. (14) through Lagrangian Relaxation by moving the slew calculation into the objective function.
and eliminate all the 0.95 \cdot Slw, because they are constants, Eq. (15) provides the corresponding slew optimization formulation, where each segment slew is multiplied with a Lagrangian Multiplier (LM), i.e. \( \beta_{ij} \), which is set to 1 as the initial value.

\[
\min \sum_{i \in P^{\text{critical}}} \sum_{j=1}^{L} \beta_{ij} \cdot Slw_c(i, j) \cdot a_{ij}, \\
\text{s.t. (7b) – (7e)}.
\]  

During each iteration, LMs are updated as shown in Eq. (16),

\[
\beta_{ij} = \beta'_{ij} \cdot \sqrt{\frac{Slw_{sink}(P^{\text{critical}})}{Slw_c}},
\]

where \( \beta'_{ij} \) is the LM in the previous iteration, and \( Slw_{sink}(P^{\text{critical}}) \) is the sink slew of critical path \( P^{\text{critical}} \). With the consideration of sink slew, we impose more weights on longer paths. Therefore, in the example of \( P^{s} \) violations, we start from the first segment on the critical path (line 4), and can obtain \( Slw \) that is assigned on a layer higher than \( order \). To find this segment, we traverse each non-critical segment \( S \) and check if there exist slew violations. For those nets with violations, they can introduce slew discrepancies by calculating the segment slew based on the previous assignments. Therefore, we implement a post slew optimization algorithm to further improve the segment slew violation (lines 14–28). Then the improvement outperforms the current most improvement, we will skip the current layer and eliminate all the \( S \).

Similar with Eq. (7), Eq. (15) is solvable through ILP because we can obtain \( Slw_{sink}(P^{\text{critical}}) \) based on the last iteration. Still, we incorporate the via capacity constraints into the objective function with the same linearization method as in Eq. (13). Ultimately, the problem can be formulated as a weighted sum of \( a_{ij}s \) and solved through min-cost max-flow model.

After solving the problem in each iteration, we update the input slew and check if there is a convergence of slew violations. If the improvement is below a certain ratio, then the slew optimization terminates. In summary, this algorithm provides a slew targeted optimization because it considers both the upstream segments and downstream segments. Meanwhile, more emphasis is placed on critical paths by taking the sink slew into accounts.

Based on the slew model, the segment input slew can affect the output slew directly, but during each iteration, we obtain the input slew of each segment based on the last iteration. Thus, it may introduce slew discrepancies by calculating the segment slew based on the previous assignments. Therefore, we implement a post slew optimization algorithm, which mainly focuses on fixing local violations while considering current layer assignments of the whole path. The details of this algorithm is given in Section III-G.

G. Post Slew Optimization

In this section, we propose a post slew optimization algorithm to further reduce the slew violations. The pseudo code is shown in Algorithm 2. Based on the global optimization results, we traverse each net sink to check if there exist slew violations. For those nets with violations, they are saved in a net set, i.e. \( N_{slw} \), and sorted in the descending order of slew violations (line 2). The net with the highest priority is the one with the most segments causing slew violations. To cope with slew violations, we start from the first segment on the critical path (line 4), and adjust the layer assignment of each segment \( s \) through two steps (lines 5–34).

First, if there exists any available routing capacity for \( s \) on higher layers (line 7) and its segment slew can be improved (line 8), we record the improvement and mark this layer as a candidate (line 9). Meanwhile, the induced via capacity violations cannot exceed a given ratio, \( Ra \). After traversing each possible layer, the layer with the most improvement is selected for \( s \) to assign (line 13). In this way, the sink slews of other nets are not affected while the current segment output slew is improved. However, if no available layer is found, a second step is required to improve the segment slew violation (lines 14–28).

In the second step, we search for a non-critical segment on the same edge with \( s \). When exchanging its layer with segment \( s \), we would not degrade its slew much while improving the output slew of \( s \). In order to find this segment, we traverse each non-critical segment \( s \) that is assigned on a layer higher than \( l(s) \) and able to bring slew improvements for \( s \) (lines 16–18). Then the slew improvement is calculated by switching the layer of segment \( s \) and segment \( s \) (line 19). If the improvement outperforms the current most improvement, we signify this segment as \( stemp \), and record its layer (lines 20–24). Here we also take into accounts the net which segment \( s \) belongs to. When its sink slew is close to the given slew constraint, then segment \( s \) will not be considered as an exchange candidate. After traversing each segment on higher layers, we switch the assigned layers of segments \( s \) and \( stemp \) and update the slews of the corresponding nets (lines 26–27). When the slew violation of \( P^{\text{critical}} \) has been fixed, then we continue to fix the next net in \( N_{slw} \) (lines 29–31). The segments of each net are traversed in a top-down manner from driver to sinks. When a segment has already exceeded the slew constraint, we will skip the remaining segments in this net because there is no further optimization space for sink slews of this net. By this way we can further reduce the runtime overhead (lines 32–34). The algorithm ends until all nets in \( N_{slw} \) are traversed. In comparison to slew optimization in Section III-F, this algorithm adjusts the layer assignment of segments based on their real input slew, thus providing a more accurate slew optimization. Meanwhile, if there are only a few slew critical nets, it is efficient to fix the violations through this algorithm.

Algorithm 2 Post Slew Optimization Algorithm

Require: Current layer assignment solution;
1: Save all slew critical nets in \( N_{slw} \);
2: Sort nets in the descending order of slew violations;
3: for each net \( n \in N_{slw} \) do
4: for each \( s \) in \( P^{\text{critical}} \) do
5: Initialize \( Slw_{imp} = 0 \);
6: for each \( l \in c(s) \) do
7: if Routing capacity exists for layer \( l \) then
8: if \( Slw_{imp}(l) \geq Slw_{imp} \) and \( OV \leq Ra \) then
9: Update \( l_{temp} \) and \( Slw_{imp} \);
else
10: end if
11: end if
12: end for
13: Assign \( s \) on \( l_{temp} \);
14: if No \( l_{temp} \) is found then
15: for each non-critical \( s \) on \( c(s) \) do
16: if \( Slw_{imp}(l(s)) \leq 0 \) then
17: Continue;
else
18: if \( \delta Slw_{imp}(l(s)) \geq \delta Slw_{imp}(l(s)) + \delta Slw_{imp}(p, l(s)) \) and \( OV \leq Ra \) then
19: Update \( stemp \) and \( Slw_{imp} \);
end if
20: end if
21: if \( Slw_{imp} \leq \alpha \cdot Slw_{c} \) then
22: Update \( stemp \) and \( Slw_{imp} \);
23: end if
24: end if
25: end for
26: Switch layers between \( s \) and \( stemp \);
27: Update \( Slw \) for \( n(s) \) and \( n(stemp) \);
end if
28: if \( Slw_{imp}(P^{\text{critical}}) \leq Slw_{c} \) then
29: break;
30: end if
31: if \( Slw_{imp}(l') \geq Slw_{c} \) then
32: break;
33: end if
34: end if
35: end for
36: end for
IV. EXPERIMENTAL RESULTS

We implemented the proposed timing-driven incremental layer assignment framework in C++, and tested it on a Linux machine with 2.9 GHz Intel® Core and 192 GB memory. We selected open source graph library LEMON [40] as our min-cost network flow solver, and utilized OpenMP [41] to provide parallel computing. In our implementation, the default K value is set to 6, and the default thread number is set to 6.

A. Evaluation on ISPD 2008 Benchmarks

In the first experiment, we evaluate our timing-driven layer assignment framework on ISPD 2008 benchmarks [42]. The NCTU-GR 2.0 [9] is utilized to generate the initial global routing solutions. The initial layer assignment results are from NVM [18], which is targeting at via number and overflow minimization. Our framework is tested the effectiveness to incrementally optimize the timing. To calculate the wire delay in Eq. (1) and via delay in Eq. (2), all the metal wire resistances, metal wire capacitances, and via resistances are listed in TABLE III. Column “C” lists the capacitance. Columns “R” list the resistances for wire layers and via layers, respectively. The resistances and capacitances of wires are directly from [11], while the via resistance values are normalized from industry settings in advanced technology nodes. Since ISPD 2008 benchmarks do not provide the input capacitance and output resistance values of sinks, here we assume they are zero.

<table>
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</table>

TABLE III: Normalized capacitance and resistance.

Critical net ratio α is a user-defined parameter to control how many nets are released to incremental layer assignment. In TABLE IV, ratio α is set to 1% and 5%. Fig. 11 analyzes the impact of ratio value to the performance of incremental layer assignment framework. Fig. 11(a) shows the impact of ratio value on the maximum delay, where we can see that the maximum delays are kept the same. This means for these test cases, releasing 1% of critical nets is enough for maximum delay optimization. Fig. 11(b) shows the impact of ratio value on the average delay, where we can see increasing the ratio value can slightly improve the average delay. Fig. 11(c) is the impact on the runtime, where we can see that the runtime increases along with the increase of ratio value. From these figures we can see that the ratio value can provide a trade-off between average delay and the speed of our tool.

Our incremental layer assignment utilizes OpenMP [41] to implement multi-threading. Fig. 12 analyzes the performance of our layer assignment framework under different partition and thread numbers. Thread 1 corresponds to 1 × 1 partition, thread 2 corresponds to 2 × 2 partitions, and so on. With more partitions, the size of network flow model is reduced quadratically thus benefiting the runtime significantly together with multi-threads. From Fig. 12(a) and Fig. 12(b) we can see that the impact of thread number on both maximum delay and average delay is insignificant. Similarly, through Fig. 12(c) we can see the impact on overflow is also negligible. From Fig. 12(d) we can observe that more thread number can achieve more speed-ups. However, when thread number is larger or equal to 6, the benefit to runtime is not clear. Therefore, in our implementation the thread number is set to 6.

To demonstrate the benefit of solving the problem in a global manner, we implement a greedy strategy to assign segments in a net-by-net manner. All the reassigned nets are sorted based on their timing priorities so that a more critical net has higher priority for higher metal resources. For each net, segments are traversed sequentially and layers are selected based on the same costs as that in min-cost max-flow network. Here we release 1% critical nets and 1% non-critical nets. The results are shown in Fig. 13. From the figure, we can observe that for both average and maximum delay TILA can achieve a little bit better results compared with the greedy method. The main reason is that the greedy methodology assigns higher priorities to those critical nets so that these nets are able to take advantage of higher layer resources. Since those nets utilize high metal layers efficiently, significant timing optimization can also be achieved through this greedy methodology. Nevertheless, they sacrifice the via capacity violations due to their preferences to high layer resources. Regarding the runtime, as shown in Fig. 13(d), due to the net-by-net scheme, the greedy method is faster than TILA. Therefore, to control timing optimization and capacity constraint in a reasonable manner, a global optimization engine is more promising.

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TABLE IV: Performance comparisons on ISPD 2008 benchmarks.

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<th>(D_{max} ) ((10^6))</th>
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</tr>
<tr>
<td>newblue5</td>
<td>0</td>
<td>94425</td>
<td>7.28</td>
<td>18987.0</td>
<td>77.43</td>
<td>103.4</td>
<td>0</td>
<td>96740</td>
<td>6.57</td>
<td>3963.7</td>
<td>78.67</td>
<td>686.8</td>
<td>105034</td>
<td>5.99</td>
<td>3964.6</td>
<td>82.39</td>
</tr>
<tr>
<td>newblue6</td>
<td>369</td>
<td>146737</td>
<td>7.01</td>
<td>13416.0</td>
<td>160.57</td>
<td>236.7</td>
<td>369</td>
<td>141936</td>
<td>5.91</td>
<td>12028.2</td>
<td>166.58</td>
<td>1213.3</td>
<td>158329</td>
<td>5.06</td>
<td>12033.0</td>
<td>183.94</td>
</tr>
</tbody>
</table>

average ratio 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.97 | 0.90 | 0.47 | 1.03 | – | 0.97 | 0.81 | 0.47 | 1.11 | –

B. Evaluation on 20nm Industry Benchmarks

In the second experiment, we test our incremental layer assignment framework on eight 20nm industry test cases (Industryly-Industry8). We called an industry tool to generate initial global routing and layer assignment solutions. Different from the preceding experiment, here we use industry resistance and capacitance values to calculate the wire delays and the via delays. TABLE V lists the details of performance evaluation, where for each method columns “OV#”, “\(D_{avg}\)”, “\(D_{max}\)”, and “viaw#” provide the overflow number, average delay, maximum delay, and total via number. Since all the critical nets are provided in the benchmarks, the critical and non-critical selection phases are skipped in this benchmark suite. We can see that

Fig. 11: Performance impact on different ratio values. (a) The impact of ratio on maximum delay; (b) The impact of ratio on average delay; (c) The impact of ratio on runtime.

Fig. 12: Evaluation thread number impact on three test cases in ISPD 2008 benchmark suite. (a) The impact on maximum delay; (b) The impact on average delay; (c) The impact on overflow; (d) The impact on runtime.
such zero overflow performance. In summary, from TABLE V we can
solution is with zero overflow, and our framework can also maintain
potential space for via counts optimization. The initial layer assignment
the via counts. Finally, industrial benchmarks provide a more even
included in our mathematical formulation, which also helps to control
via counts. Finally, industrial benchmarks provide a more even
layer assignment of segments through all the layers. This provides us a
potential space for via counts optimization. The initial layer assignment
solution is with zero overflow, and our framework can also maintain
such zero overflow performance. In summary, from TABLE V we can see our incremental layer assignment framework can achieve significant
timing improvement.

C. Slew Comparisons on ISPD & 20nm Industry Benchmarks
In this section, we compare TILA with slew optimization (TILA-S)
against TILA without slew improvement (TILA). Still, the effectiveness is
verified by both ISPD and industry benchmarks with slew constraints.
For ISPD benchmarks, the problem sizes are so different that one
single constraint is not applicable to all benchmarks. Thus, we set the
slew constraint of each benchmark as 5 times its initial average delay
as shown in TABLE IV. In this manner, the initial number of slew
violations is in proportion to the number of total segments for each
benchmark. However, the slew constraints for industry benchmarks are
given based on industrial settings.

TABLE VI lists the results for ISPD benchmarks by comparing
TILA-S-1% with TILA-1% while releasing 1%. Besides the performance
metrics shown in TABLE IV, we introduce an additional column
“SV#” which gives the number of slew violations, and the second
column lists the initial number of violations. TILA-1% provides the
intermediate results after delay optimization, while TILA-S-1% shows
the final results. We can see that TILA-1% is able to reduce the slew
violations significantly from $6.89 \times 10^4$ to $3.57 \times 10^4$, because delay
optimization also benefits slew violations considering the downstream
segments. However, with the slew targeted optimization, this number
can further be reduced by 48%. Meanwhile, average delay also
decreases by 2%, which shows that slew optimization can also benefit
delay slightly. The maximum delay keeps similar with TILA, because
its optimization space is limited after delay optimization. For vias and
violations, there is no obvious difference between TILA-S and TILA.
The main penalty of TILA-S is the 69% increase of runtime due to
additional two-stage slew optimization. Based on the results, we observe
that TILA-S can handle slew violations efficiently while keeping similar
delay and via performance.

Fig. 14 shows the effect of adopting post slew optimization for some
small cases of ISPD 2008 benchmarks. It is shown that the post slew
optimization stage improves the number of slew violations slightly

<table>
<thead>
<tr>
<th>Bench</th>
<th>Industry Layer Assignment</th>
<th>TILA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OV#</td>
<td>$D_{avg}$</td>
</tr>
<tr>
<td>Industry1</td>
<td>0</td>
<td>6204.6</td>
</tr>
<tr>
<td>Industry2</td>
<td>0</td>
<td>6049.6</td>
</tr>
<tr>
<td>Industry3</td>
<td>0</td>
<td>6025.4</td>
</tr>
<tr>
<td>Industry4</td>
<td>0</td>
<td>5572.8</td>
</tr>
<tr>
<td>Industry5</td>
<td>0</td>
<td>5531.4</td>
</tr>
<tr>
<td>Industry6</td>
<td>0</td>
<td>5443.5</td>
</tr>
<tr>
<td>Industry7</td>
<td>0</td>
<td>5066.0</td>
</tr>
<tr>
<td>Industry8</td>
<td>0</td>
<td>4096.4</td>
</tr>
<tr>
<td>average</td>
<td>0</td>
<td>5514.9</td>
</tr>
<tr>
<td>ratio</td>
<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>
TABLE VI: Comparisons on ISPD 2008 benchmarks for slew optimization.

<table>
<thead>
<tr>
<th>Bench</th>
<th>NVM [18]</th>
<th>TILA-1%</th>
<th>TILA-S-1%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SV#</td>
<td>SV#</td>
<td>VO#</td>
</tr>
<tr>
<td>adaptec1</td>
<td>8.57</td>
<td>4.59</td>
<td>50716</td>
</tr>
<tr>
<td>adaptec2</td>
<td>24.75</td>
<td>10.38</td>
<td>36824</td>
</tr>
<tr>
<td>adaptec3</td>
<td>19.77</td>
<td>8.22</td>
<td>89800</td>
</tr>
<tr>
<td>adaptec4</td>
<td>54.23</td>
<td>16.05</td>
<td>67946</td>
</tr>
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<td>adaptec5</td>
<td>54.65</td>
<td>21.35</td>
<td>81956</td>
</tr>
<tr>
<td>bigblue1</td>
<td>16.68</td>
<td>8.12</td>
<td>46151</td>
</tr>
<tr>
<td>bigblue2</td>
<td>81.77</td>
<td>59.00</td>
<td>114215</td>
</tr>
<tr>
<td>bigblue3</td>
<td>67.42</td>
<td>38.06</td>
<td>65437</td>
</tr>
<tr>
<td>bigblue4</td>
<td>118.28</td>
<td>67.48</td>
<td>98987</td>
</tr>
<tr>
<td>newblue1</td>
<td>46.67</td>
<td>36.60</td>
<td>56602</td>
</tr>
<tr>
<td>newblue2</td>
<td>62.98</td>
<td>29.76</td>
<td>33941</td>
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<td>52.56</td>
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<td>84273</td>
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<tr>
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<td>70.99</td>
<td>151300</td>
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<td>newblue6</td>
<td>88.69</td>
<td>49.83</td>
<td>96740</td>
</tr>
<tr>
<td>newblue7</td>
<td>181.17</td>
<td>89.48</td>
<td>141936</td>
</tr>
</tbody>
</table>

| average ratio | 68.91 | 35.69 | 81122 | 5.92 | 9082.9 | 54.93 | 406.8 | 18.68 | 80758 | 5.82 | 9069.7 | 55.68 | 670.1 |

Fig. 14: Comparison between with and without post slew optimization stage on some small test cases: (a) on average delay; (b) on maximum delay; (c) on slew violations.

Fig. 15: Convergence with iteration number of TILA-S on some small test cases: (a) on average delay; (b) on slew violations.

without affecting average delay and maximum delay. The main reason is that during selection of switching candidate segments, we take its current slew into consideration. Once the candidate is selected with the smallest slew degradation, its impact on delay is also negligible because slew is closely related with delay.

To illustrate the timing convergence of our iterative framework, we relax the convergence constraint for delay and slew optimization, and record the average delay and slew violation number for each iteration till the fifth iteration. Fig. 15 shows the timing convergence with iteration number. The 0-th iteration corresponds to the initial solution, where we can see a clear convergence after first two iterations.

As stated in Section III-F and Section III-G, our slew optimization flow reduces the number of slew violations and benefits the buffering overhead. To make this explicit, we measure the number of buffers we may adopt for each ISPD 2008 benchmark in Fig. 16. Here we implement a top-down algorithm to insert buffers in a net-by-net manner. For each net with slew violations, we traverse from its driver and insert one buffer when there is a slew violation; meanwhile, we assume the input slew of each net and the output slew from the buffer are both equal to 0. After traversing one net, we can obtain the number of buffers used in this net to fix the violations. It is shown that the average buffering overhead can be reduced from 9258 to 7586 in Fig. 16. Therefore, our post slew-targeted optimization helps to reduce the buffering overhead, and is also able to provide an estimate of buffering overhead at pre-buffering stage.

For the 20nm industry benchmarks, besides delay and via metrics, we also take slew violations into account. TABLE VII shows that...
TABLE VII: Comparisons on 20rmi industry benchmarks for slew optimization.

<table>
<thead>
<tr>
<th>Bench</th>
<th>SV#</th>
<th>TILA</th>
<th>TILA-S</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DAVG</td>
<td>DMAX</td>
<td>via#</td>
</tr>
<tr>
<td>Industry1</td>
<td>24</td>
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<td>28667.2</td>
</tr>
<tr>
<td>Industry2</td>
<td>18</td>
<td>3796.4</td>
<td>27416.3</td>
</tr>
<tr>
<td>Industry3</td>
<td>10</td>
<td>3906.2</td>
<td>23820.8</td>
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<td>Industry4</td>
<td>7</td>
<td>3669.2</td>
<td>25858.9</td>
</tr>
<tr>
<td>Industry5</td>
<td>5</td>
<td>3799.3</td>
<td>34347.0</td>
</tr>
<tr>
<td>Industry6</td>
<td>3</td>
<td>3692.9</td>
<td>33096.3</td>
</tr>
<tr>
<td>Industry7</td>
<td>8</td>
<td>3693.7</td>
<td>29348.7</td>
</tr>
<tr>
<td>Industry8</td>
<td>8</td>
<td>3040.2</td>
<td>20137.7</td>
</tr>
<tr>
<td>average ratio</td>
<td>7.4</td>
<td>3650.6</td>
<td>29637.9</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The table shows the comparisons on 20 rmi industry benchmarks for slew optimization. The columns represent the benchmarks, average DAVG, DMAX, via#, and CPU(s) for TILA and TILA-S. The results indicate that TILA-S generally outperforms TILA in terms of DAVG, DMAX, and CPU(s) for most benchmarks, showing improved performance in slew optimization.

V. CONCLUSION

In this paper, we have proposed a set of algorithms to the timing-driven incremental layer assignment problem while mitigating slew violations. At first, the mathematical formulation is given to search for optimal total wire delays and via delays. Then Lagrangian relaxation based method is proposed to iteratively improve the timing of all the nets. The Lagrangian relaxation subproblem (LRS) is modeled through min-cost flow model to provide effective integral solutions. In addition, multiprocessing of K × K partitions of the whole chip provides runtime speedup. Then we integrate the slew violation optimization method into our framework to mitigate the violations. Our incremental layer assignment tool with/without slew optimization, TILA-S, is verified in both ISPD 2008 and industry benchmark suites, and has demonstrated its effectiveness. In our current implementation, slew improvement is achieved through a separate stage with delay optimization. As a future work, we plan to consider layer assignment targeting at delay and slew optimization concurrently while reducing buffer overhead. As in emerging technology nodes, the routing algorithm should be able to adapt the heterogeneous layer structures, we believe this paper will stimulate more research for timing improvement in advanced routing, and shed more light on traditional EDA topics.

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REFERENCES


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He has received a number of awards, including the SRC 2013 Technical Excellence Award, DAC Top 10 Author in Fifth Decade, DAC Prolific Author Award, ASPDAC Frequently Cited Author Award, 13 Best Paper Awards and several international CAD contest awards, Communications of the ACM Research Highlights (2014), ACM/SIGDA Outstanding New Faculty Award (2005), NSF CAREER Award (2007), SRC Inventor Recognition Award three times, IBM Faculty Award four times, UCLA Engineering Distinguished Young Alumnus Award (2009), and UT Austin RAISE Faculty Excellence Award (2014).