Accurate Waveform Modeling using SVD with Applications to Timing Analysis

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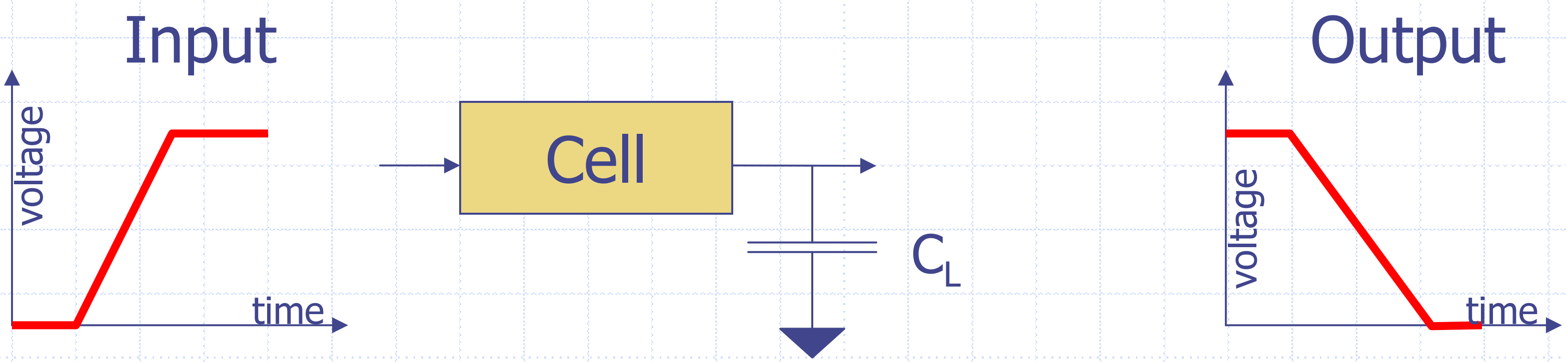
Motivation

- On-chip waveforms do not look like "ramps"
  - Devices are operating in more complex regimes and do not at all look like current sources
  - Loads, both interconnect as well as gate inputs, are resistive and non-linear
- We persist in trying to fit an outdated waveform model onto far more complicated behaviors
  - Applications like Statistical Static Timing Analysis (SSTA) require accurate modeling
  - Model inaccuracy must be « expected variability to reliably estimate performance variability

Ramp Based Timing Model

- Expressed in terms of a Ramp approximation of input and output waveforms.
  - Arrival time, Slope and Load Capacitance



$V_i = f(t, t_i, s_i)$ $V_o = f(t, t_o, s_o)$

Timing Model: $[t_o, s_o] = F([t_i, s_i], C_L)$

Source of Error in Timing Models

1. Inability of the waveform function (ramp) to fit the real waveform.
2. Estimation of a complex load by a single capacitance.
3. Lack of complete modeling support (coupling noise, multiple input switching etc…).

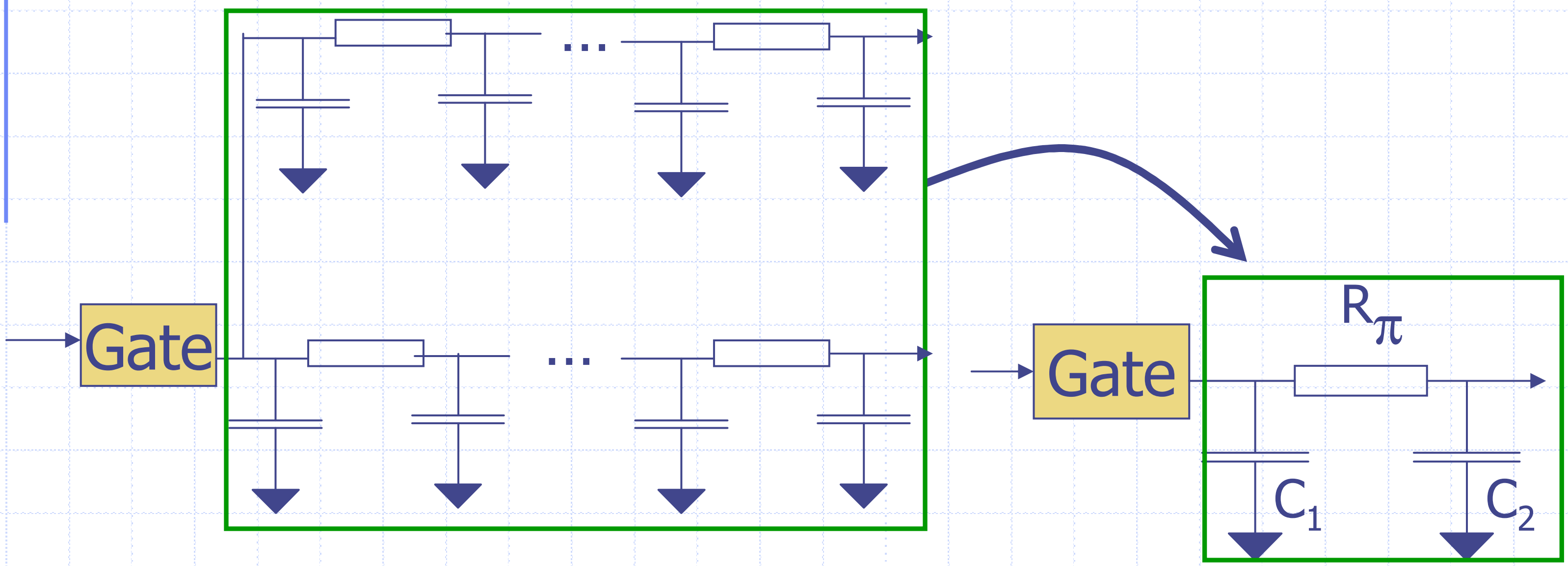
- We are focusing in this paper on the first two sources of error:
  - Part 1: π-model for interconnect
  - Part 2: Accurate waveform modeling

Part 1: Better Load Modeling

- Benefits:
  - A π model of the load is clearly a better representation than a single capacitor.
  - We did not do a complete study to exactly quantify the improvement achieved.
- Costs:
  - Modeling a gate's behavior as a function of a π load means we have 2 more variables to vary.
  - When using traditional (e.g. full factorial) experiment designs to create the timing models, adding 2 variables can be quite costly.

Existing Work on π-Models

- O'Brien and Savarino developed an algorithm for reducing an RC tree to a driving-point π–model.



Solving the Dimensionality Problem

- A naïve implementation of a gate model builder may use a full factorial design, resulting in an exponential number of simulation vs. modeling variables.

- We use Latin-Hypercube Sampling, a well established statistical sampling technique instead.
  - Number of simulation ~ linear in number of modeling variables.

Part 2: Accurate Waveform Modeling

History:

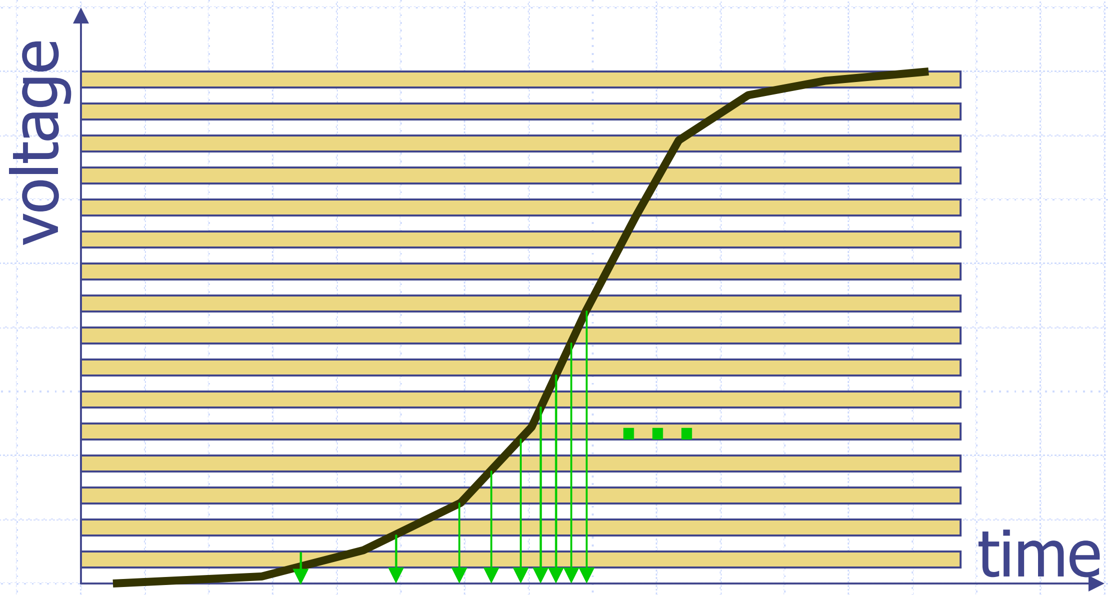
- Heuristic models
  - Equivalent waveform model [Hashimoto, ICCAD '03]
  - Weibull distribution [Amin, ICCAD '03]
- Change of basis models
  - Model current not voltage, CSM [Amin, DAC '06]
- Data based models
  - Basis decomposition [Jain, ICCAD '05]
  - PCA based approach [Nassif, TAU '04]

Our approach: extend the PCA approach

- Use more appropriate SVD instead of PCA
- Generate waveform model based on complete library
- Demonstrate application to interconnect as well

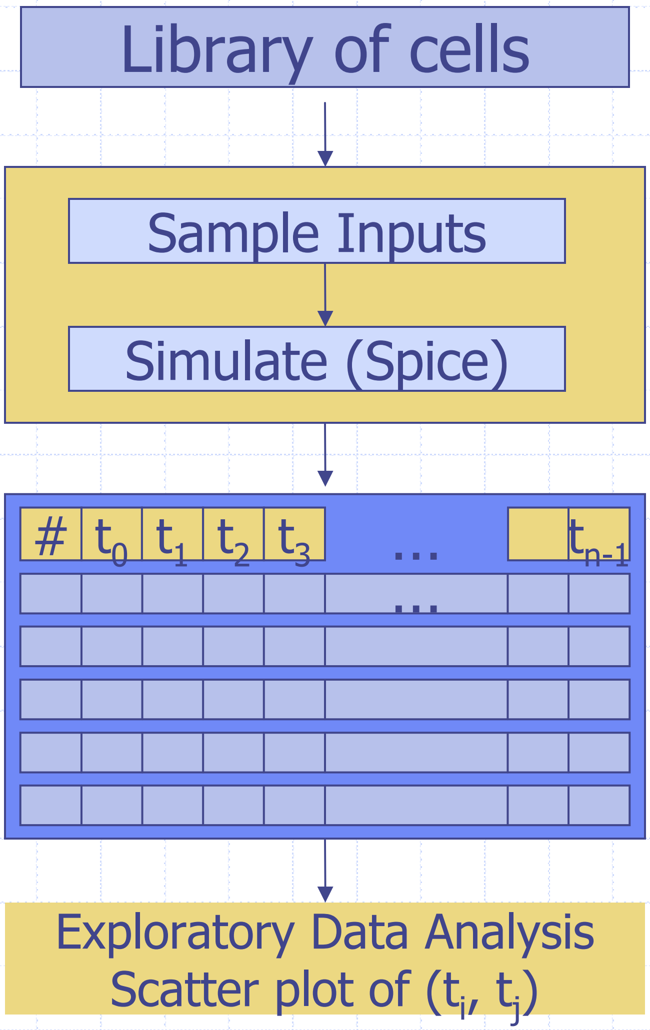
Data based model

- Divide $[0 \ldots V_{dd}]$ into n intervals.
- Measure t_0 through t_{n-1} for waveforms of interest.
  - t_i = time at which waveform crosses $V_{dd} \times (i/(n-1))$

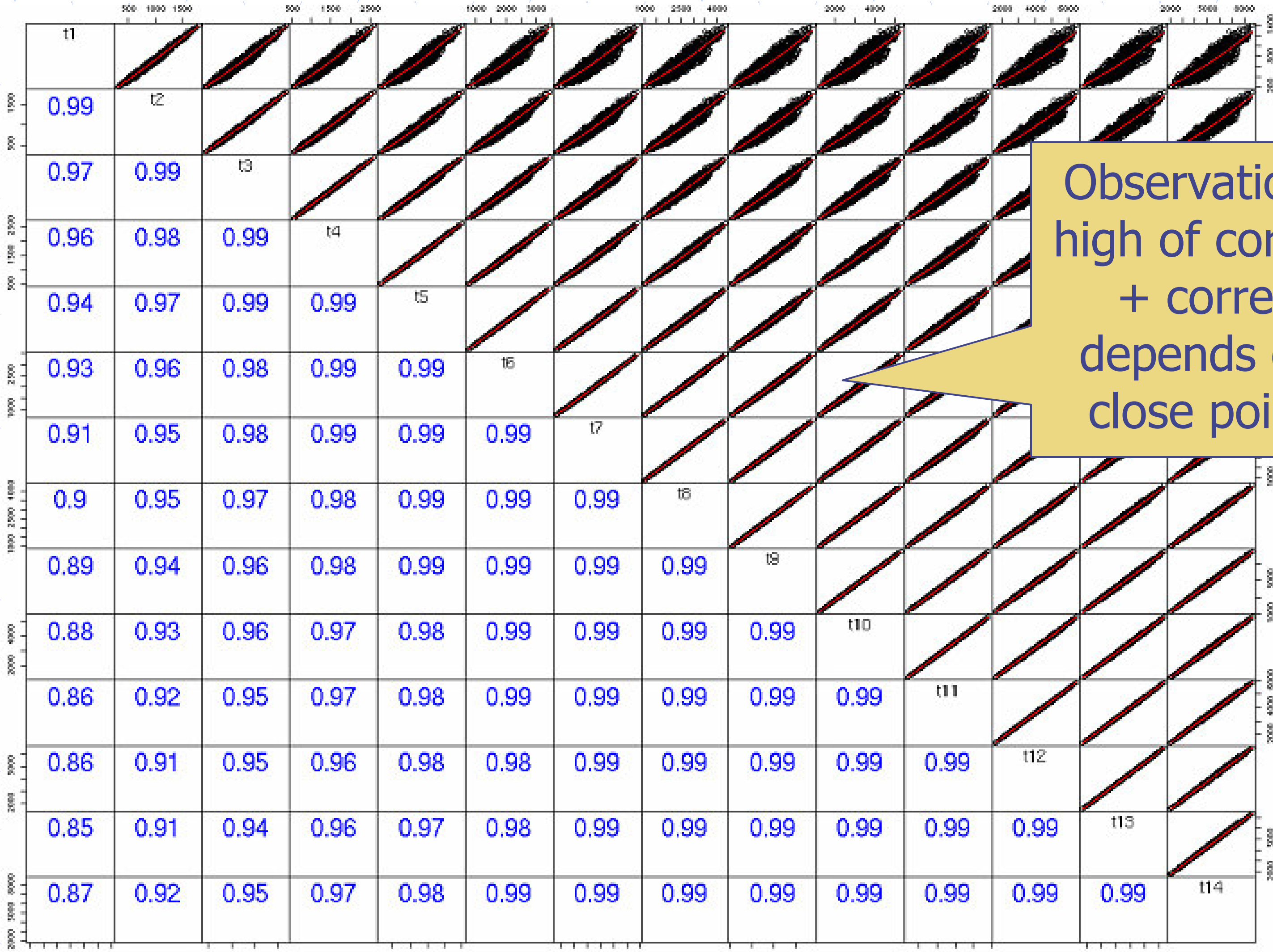


Intuition for our work

- The time/voltage (t_i, v_i) pairs that define a waveform are not independent of each other
- To verify this we analyzed waveforms obtained from various cells in the library under varying input and interconnect load conditions
  - We expect the crossing times (t_i) of these waveforms to be inter-related.



Scatter plot of (t_i,t_j) pairs

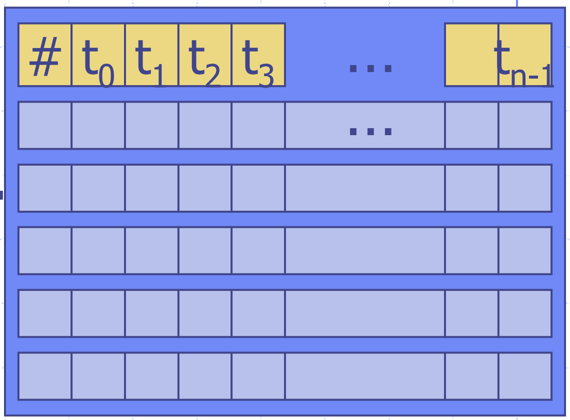


Observation: Very high of correlation, + correlation depends on how close points are

Comments on Crossing Time Stats

- The crossing times (t_i) are obviously not independent of each other
  - Strong correlation across all the crossing times
  - Then t_i can be expressed as a function of a smaller number of independent variables
- How to find a smaller subset of independent variables?
  - Previous work used PCA, which works best when the distribution of the t_i is Gaussian (not the case in general)
- We use an alternative dimension reduction technique, Singular value decomposition (SVD)
  - Designed for the more general case where the t_i are simply linearly related.

Singular Value Decomposition (SVD)



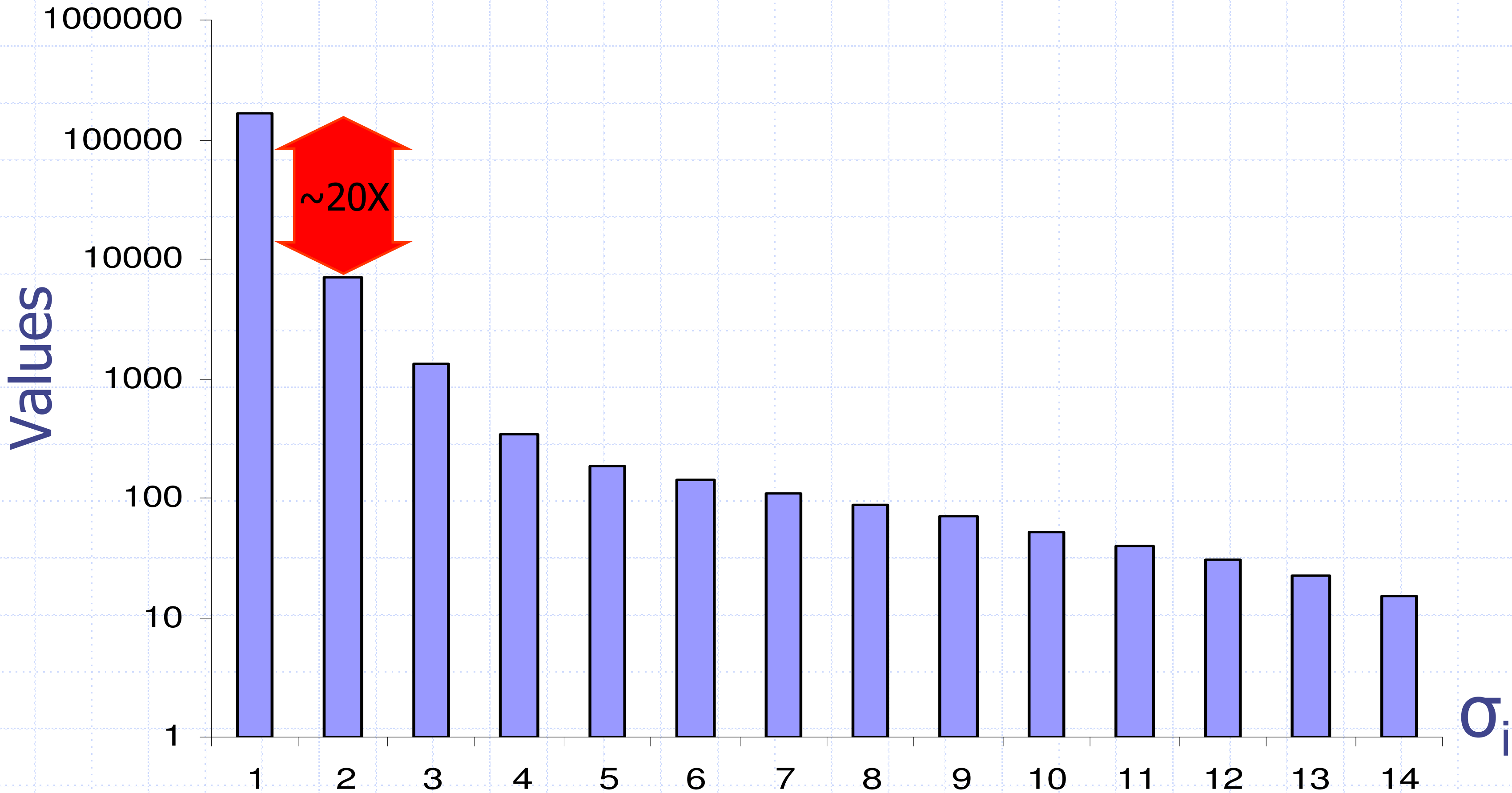
- SVD of a matrix **T** is given by

$$\mathbf{T} = \mathbf{U}\,\Sigma\,\mathbf{V}'$$

- **U** – *orthonormal* basis for columns in **T**
- **V** – *orthonormal* basis for the rows in **T**
  - Thus the *basis for waveforms*
  - Note that the basis are obtained from the *data*
  - Thus, data speaks for itself (no assumptions needed)
- **Σ** – diagonal matrix contains singular values
  - Singular values are ordered in a *non-decreasing* order
  - Singular values σ_i "weighs" the basis columns $\mathbf{V}_{\cdot i}$
  - First few basis are sufficient to capture the data (i.e. the waveform) accurately

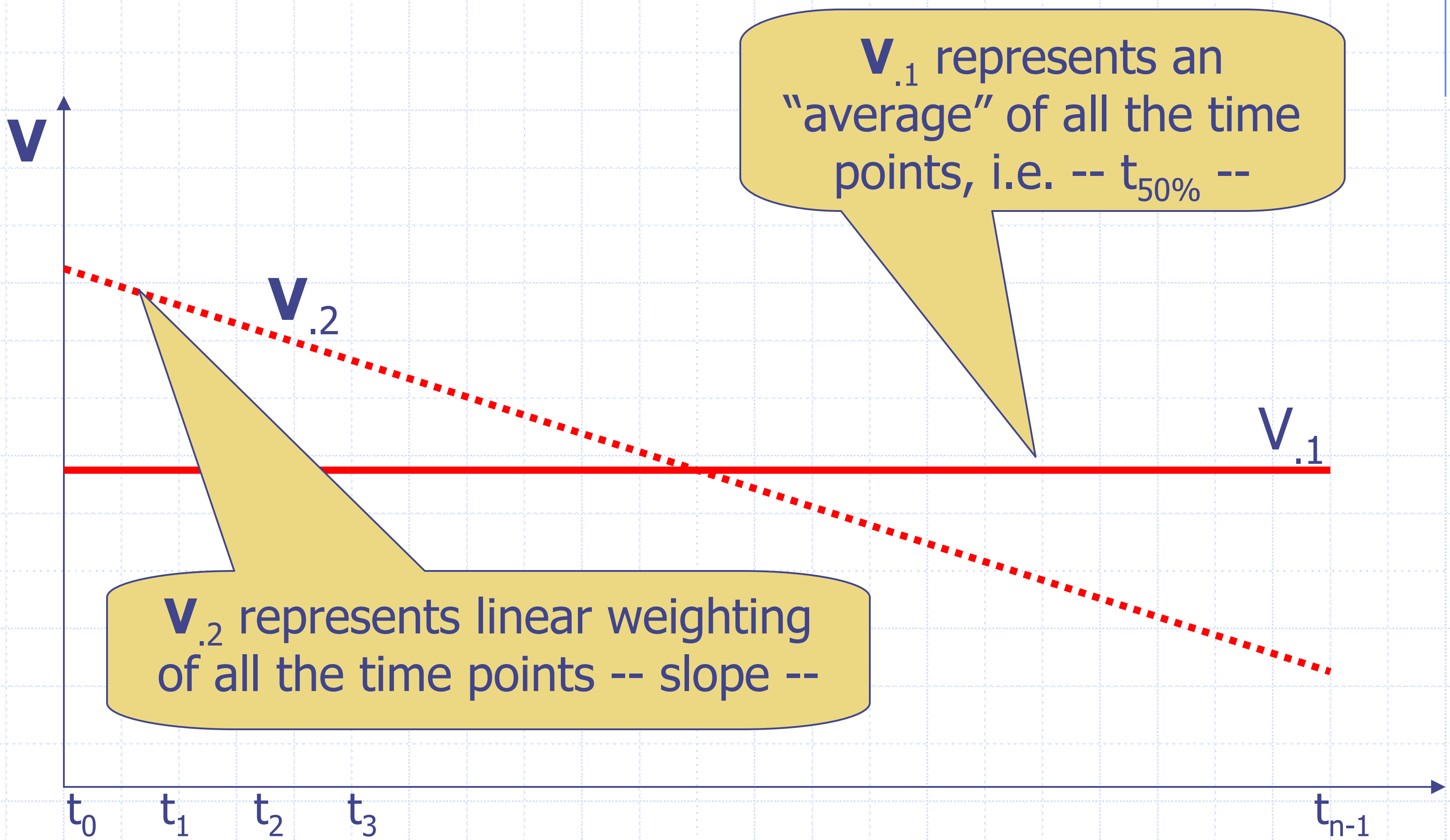
Singular values of a Matrix

- A plot of singular values of **T**
  - Only the first few dominate



Interpreting the basis vectors (**V**)

- Each of the columns of **V** represents a weighted sum of the times t_i



Example

- If the *n* time points of a *waveform* are represented as pairs ($voltage_j$,t_j)

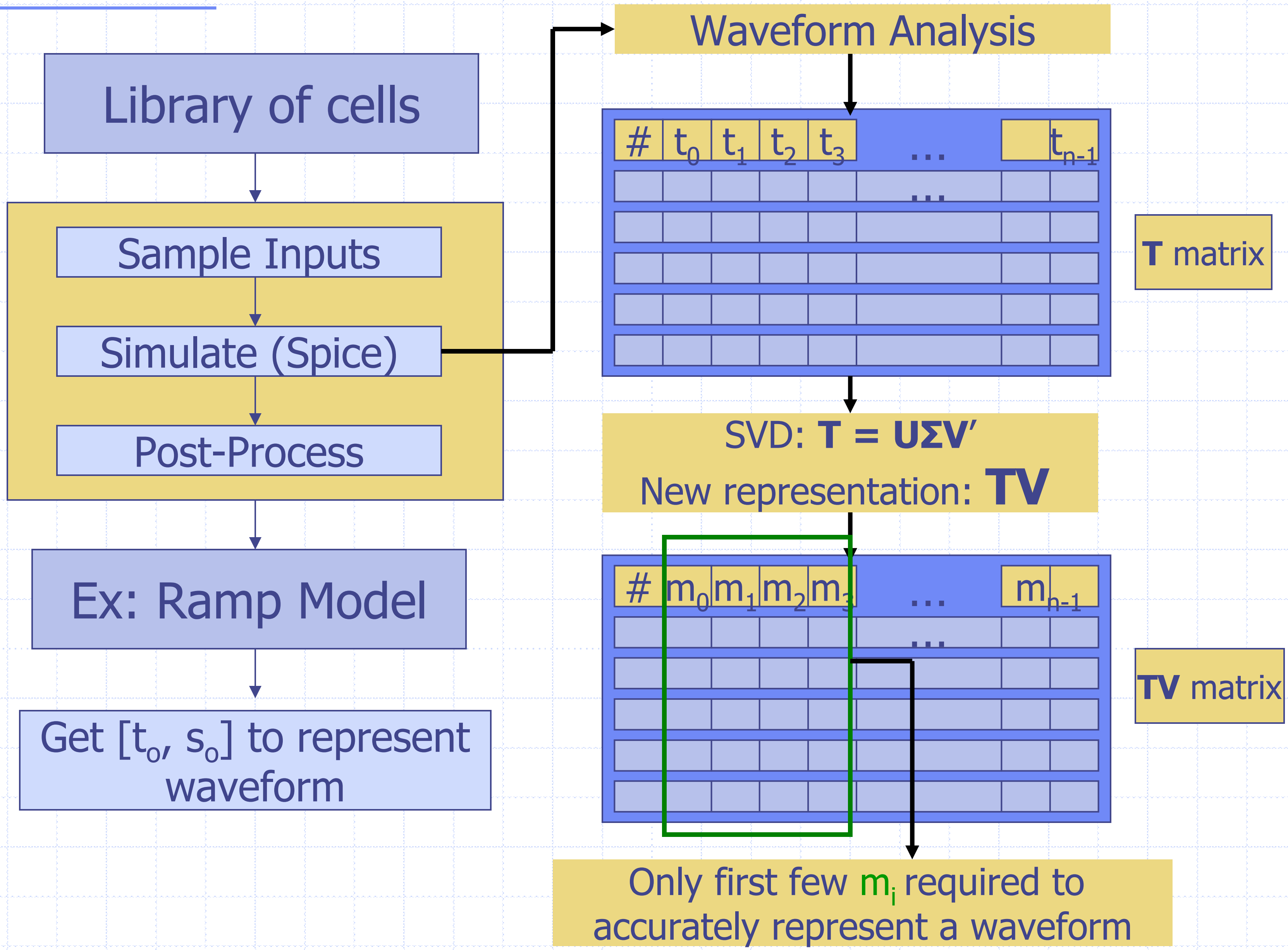
Voltage	0	1/13	…	12/13	1
t [ps]	599	632	…	788	802

- Consider $\mathbf{V}_{.2}$, which we interpreted as ~ slope in the previous slide

$\mathbf{V}_{.2}$	$v_{1,2}$	$v_{2,2}$	…	$v_{13,2}$	$v_{14,2}$
value	-0.52	-0.41	…	0.31	0.39

- The value in the new basis is given by dot product $\langle t,\mathbf{V}_{.2}\rangle$ = -202.31, which approximates the slope.
  - We call this dot product a *moment* (m_2)

Summary of SVD Analysis

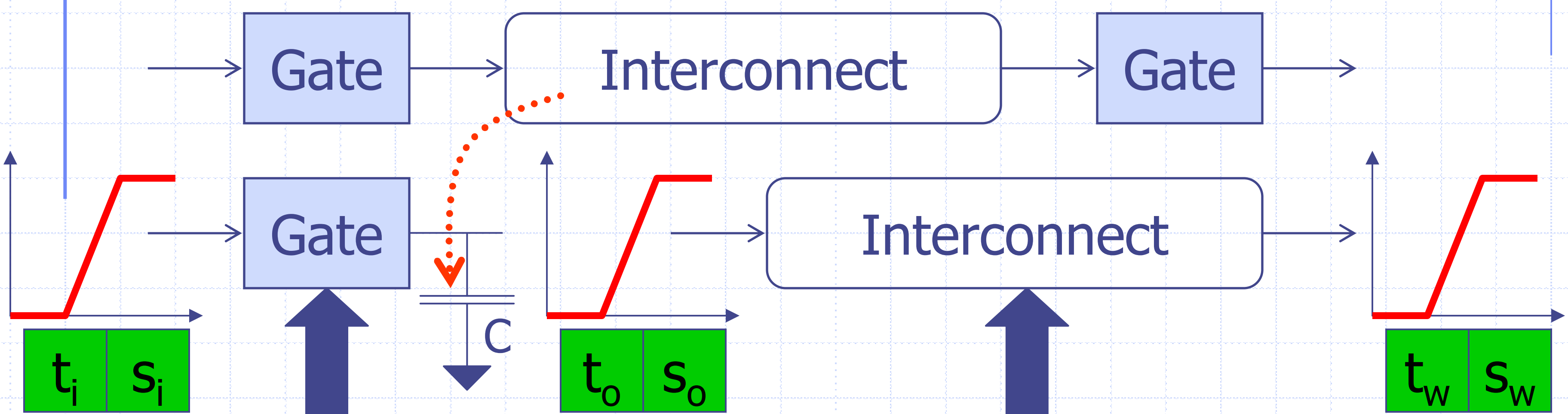


What Does This All Mean?

- It is possible to sample, simulate and analyze the set of waveforms that a *library of cells* would produce
  - From this we can determine precisely how many independent variables are required in order to represent waveforms with a specified accuracy.
  - When we analyze the entire library we might need more independent variables than for a given cell.
- Once the independent variables are selected, we also get a transformation that allows us to go from the independent variables to the waveform
  - So the complete waveform can be readily re-generated from the values of those variables.

How Does it Get Used

- In normal (ramp-based) STA, we propagate waveforms through gates and through wires.



Gate Timing Model:
$[t_o, s_o] = F_G([t_i, s_i], C_L)$

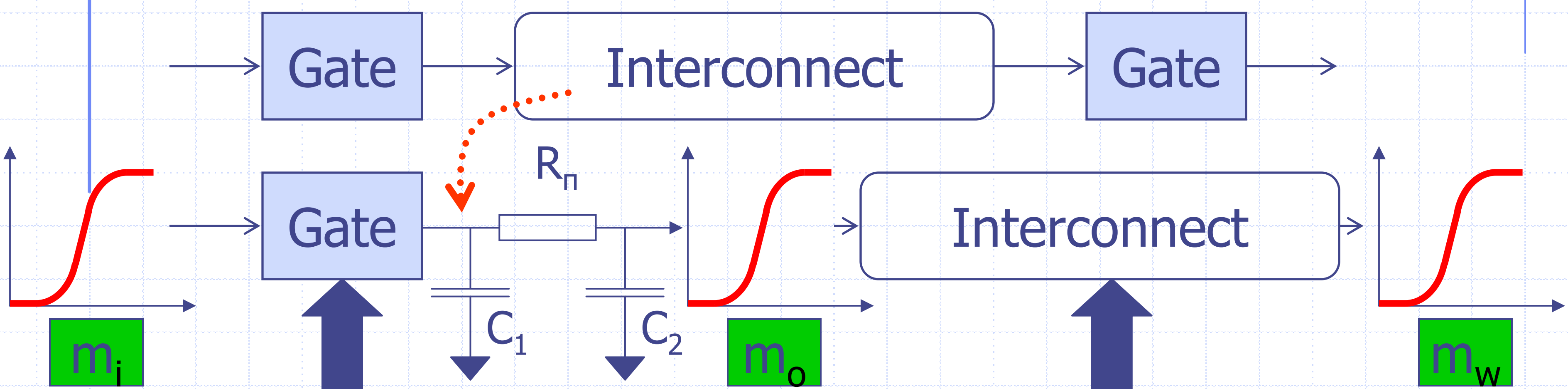
Wire Timing Model:
$[t_w, s_w] = F_W([t_o, s_o], C_L)$

Implemented using standard timing models

Implemented using tools such as RICE

SVD Models in STA

- We need to propagate waveforms through gates and through wires.



Gate Timing Model:
$[m_o] = F_G([m_i], C_L)$

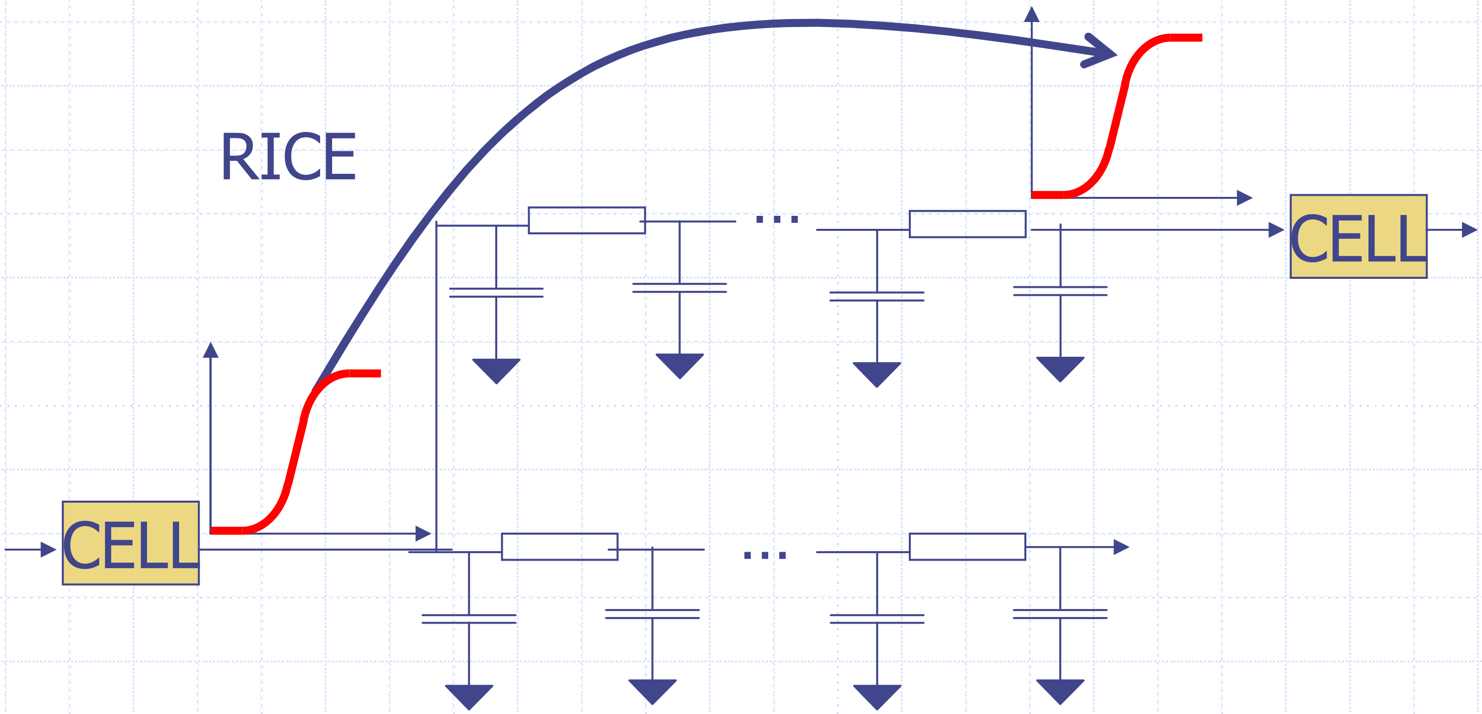
Wire Timing Model:
$[m_w] = F_W([m_o], C_L)$

Implemented using enhanced timing models

We will explore this next

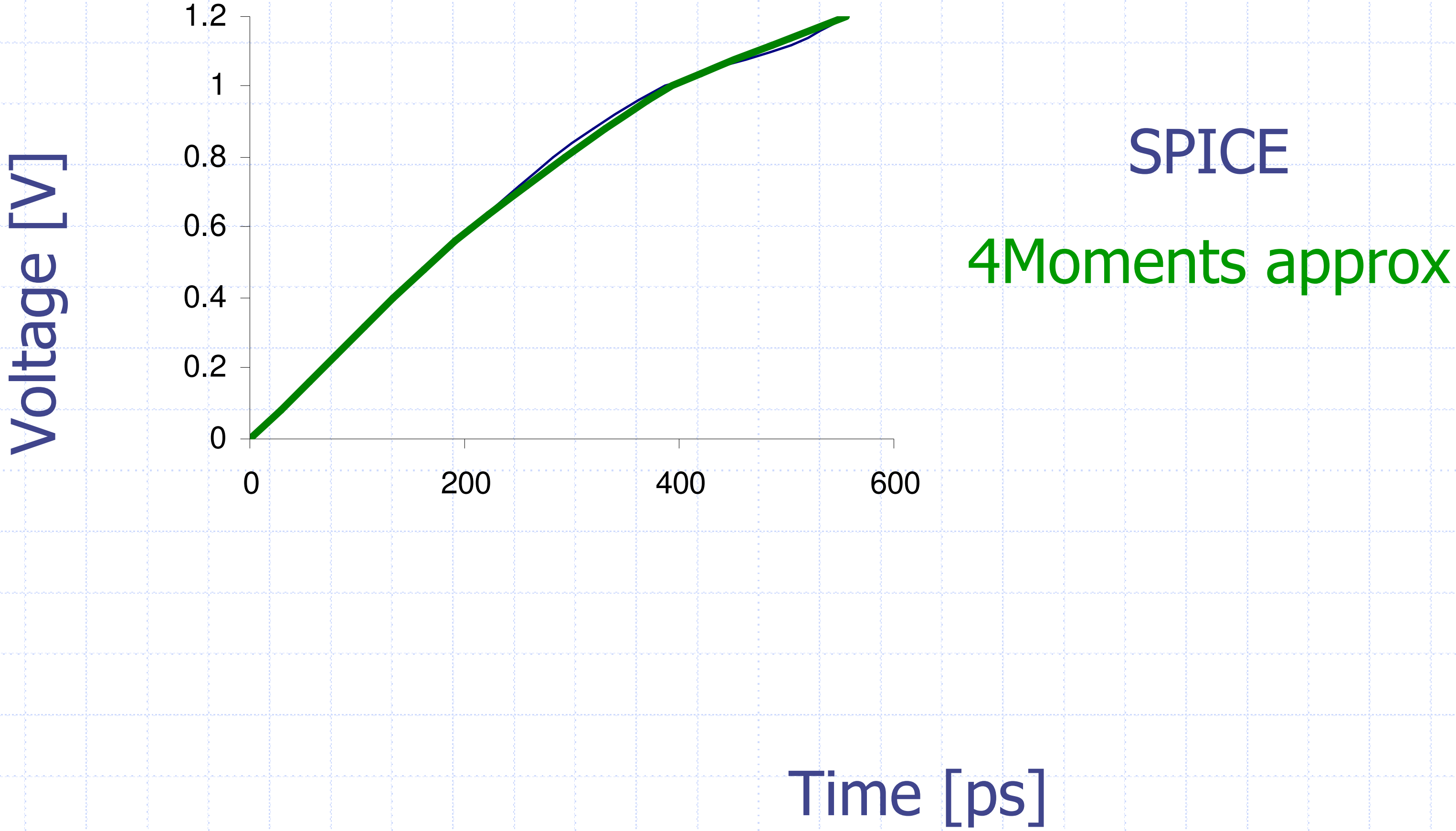
SVD + STA (Interconnects)

- We will still use RICE to propagate the waveform
- We will use the SVD transforms to convert back and forth between real waveform representation (voltage vs. time) and moments!



Waveform recovery from moments

- Waveform recovered at the far end of the interconnect of an inverter



Where Is This Model Needed

- Any time that an interface between the analog and digital world is required.
  - Input/Output from wire loads.
- Any time that knowledge of the waveform details is desired
  - SSTA, where model inaccuracy must be « expected variability to reliably estimate performance variability
  - Another Example: Estimating I_{DD} in power grid simulation
    - A ramp waveform estimate not useful for since it makes the current look like a step

Features of our model

- Precisely quantify the error we commit in modeling waveforms
- A model which can gracefully expand to model additional effects
  - Resistive wires, process variations, …
- A model which is a natural extension of the existing models
  - Allows us to use existing models where possible

Conclusions

- Advent of SSTA is causing a re-examination of how cell delay models are generated.
  - Additional dependencies are required.
  - More accuracy is needed.
- Empirical enhancements are costly in development time.
- A data-driven approach which re-uses existing data to drive improvement has the best chance of success.