Accurate Thermal Analysis Considering Nonlinear Thermal Conductivity

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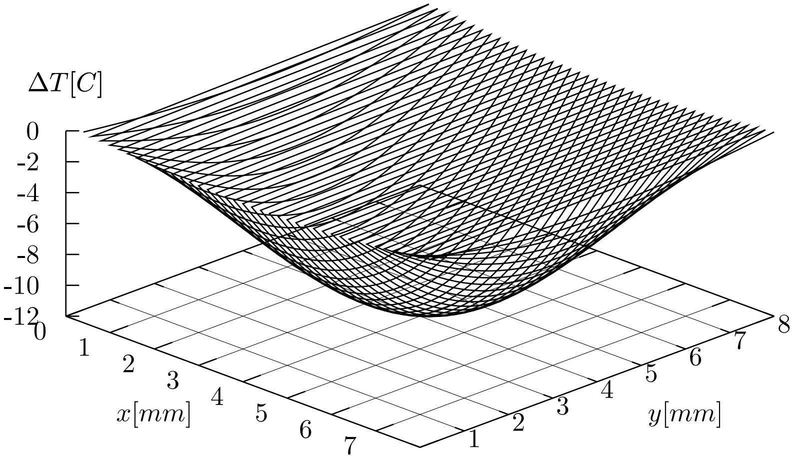
Outline

Introduction & Motivation

- Why thermal analysis?
  - Temperature affects performance
  - Temperature and power are tightly coupled
  - Reliability issues
- Is *Electrothermal* analysis any different in *digital* domain compared to *analog* domain?
  - Essentially involves solving $A(x)x = b$
  - The **size** of A is the biggest challenge

Difference in temperature on ignoring thermal conductivity



Heat conduction equation

- A partial differential equation of the form,

$$\rho C_p \frac{\partial T(x,y,z,t)}{\partial t} = \kappa(T(x,y,z,t))\,\nabla^2 T(x,y,z,t) + h(x,y,z,t)$$

  where
  - $\kappa(T(x,y,z,t))$ is the thermal conductivity
  - $h(x,y,z,t)$ is a heat source at (x,y,z,t).
  - ρC_p is the heat capacity
- Since we are interested in *steady state*, the above equation reduces to,

$$\kappa(T(x,y,z))\,\nabla^2 T(x,y,z) + h(x,y,z) = 0$$

Finite difference in 1-dimension

- Let us consider the problem in one dimension and in one layer (silicon) to simplify things The heat conduction equation reduces to,

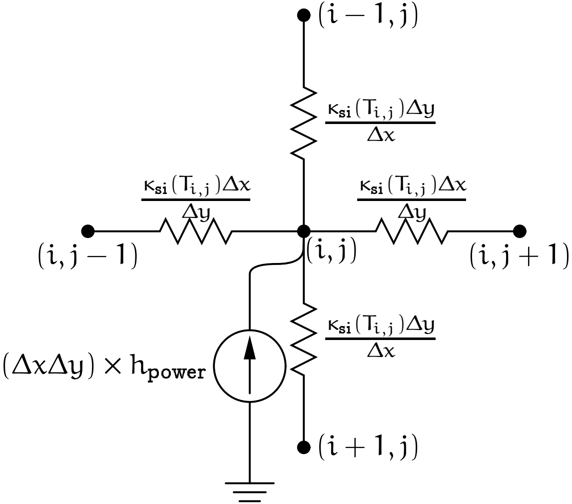
$$\kappa_{si}(T)\frac{\partial^2 T(x)}{\partial x^2} + h(x) = 0$$

- Applying finite difference to the heat conduction equation,

$$\begin{aligned}
\kappa_{si}(T_i)\left(\frac{\left(\frac{T_{i+1}-T_i}{\Delta x}\right)-\left(\frac{T_i-T_{i-1}}{\Delta x}\right)}{\Delta x}\right)+h(x) &= 0 \\
\frac{\kappa_{si}(T_i)}{(\Delta x)^2}(2T_i - T_{i-1} - T_{i+1}) &= h(x)
\end{aligned}$$

- Can be written in matrix form as $\mathbf{K}(\mathbf{T})\mathbf{T} = \mathbf{h}$.

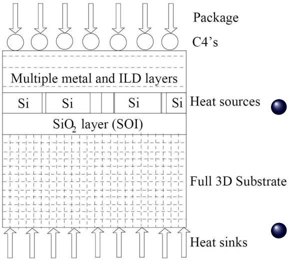
Electrical Interpretation in 2-dimension



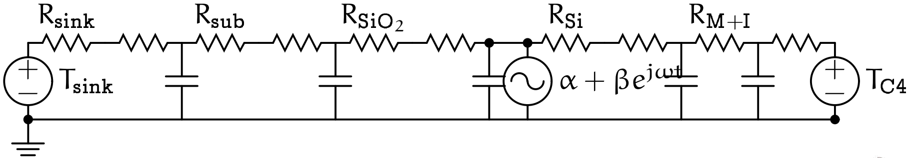
Size of the Matrix

- In literature, $\mathbf{K}(\mathbf{T})\mathbf{T} = \mathbf{h}$ is solved as $\mathbf{K}\mathbf{T} = \mathbf{h}$ [Cheng 1998, Wang 2003]
  - Linear system of equations
  - Still difficult to solve due to sheer size
  - Consider a $8000\mu m \times 8000\mu m$ chip with grid size of $\Delta x \times \Delta y = 10\mu m \times 10\mu m$
    - The size of the matrix K is $\mathbb{R}^{640,000 \times 640,000}$
    - Fortunately, the matrix is sparse
- To get an accurate solution need to solve the nonlinear system of equations $\mathbf{K}(\mathbf{T})\mathbf{T} = \mathbf{h}$
  - Very hard since there are no black box methods to solve a nonlinear system of equations

Steady state thermal circuit

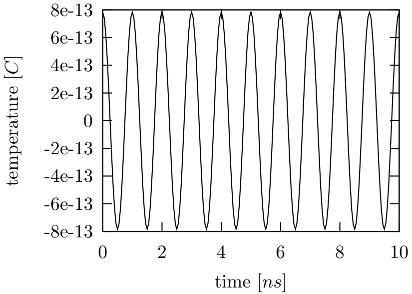


- We study if steady state (SS) heat sources can be modeled as a DC source having the RMS value of SS
- The steady state problem is studied in z direction, assume uniformity in x, y directions
- Picture of chip layers from [Su, ISLPED 2003]

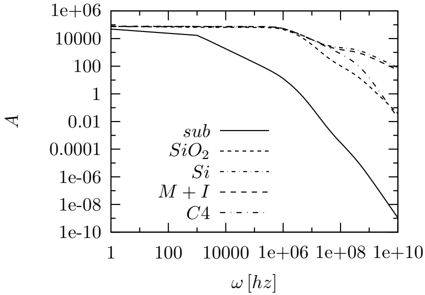


Steady State Response

- The RMS analysis is done by setting the DC sources to zero and setting the ac source to the RMS value
- The temperature rise at the substrate calculated using the RMS values for the ac sources is $2.65°$ C
- The steady state response turns out to be zero!
- Thus the approximation is good



Time constant of the substrate



- The thermal time constant is dominated by the substrate which is around Khz and the operating frequency is Ghz

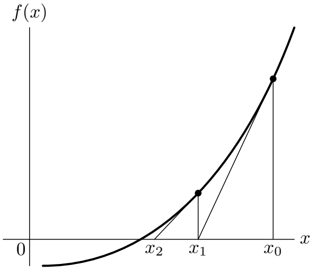
Nonlinear thermal conductivity

- Thermal conductivity is a nonlinear function of temperature
  - There is a 22 % change in thermal conductivity of Silicon over the range of $[27, 80]^{\circ}C$
  - Thermal gradients calculated assuming constant thermal conductivity is not very accurate
- Challenge is designing efficient algorithms to solve system of m nonlinear equations simultaneously

Algorithms to solve nonlinear equations

- **Newton-Raphson**

$$\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} - [\mathbf{J}(\mathbf{x}^{(k-1)})]^{-1}\mathbf{f}(\mathbf{x}^{(k-1)})$$

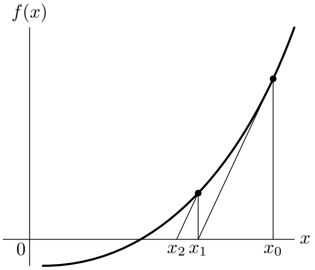


- Simple and fast convergence
- Need to factorize the Jacobian matrix during every iteration

Algorithms to solve nonlinear equations

- **Constant Jacobian**

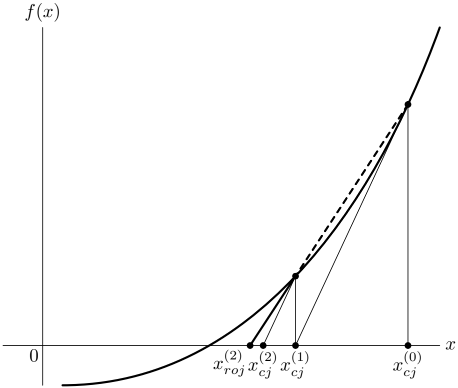
$$\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} - [\mathbf{J}(\mathbf{x}^{(0)})]^{-1}\mathbf{f}(\mathbf{x}^{(k-1)})$$



- Only *one* factorization
- But slow convergence

Sketch of Proposed Algorithm

- Accelerate **Constant Jacobian**



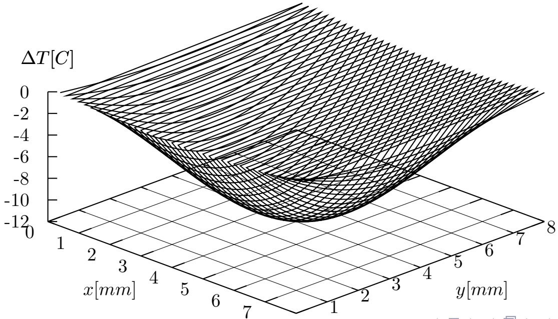
Proposed Algorithm

Input: $\mathbf{F}(\mathbf{x}) \triangleq \mathbf{A}(\mathbf{x})\mathbf{x} - \mathbf{b}$: m nonlinear equations in m unknowns
Input: p, the number of partitions $\mathbf{A}(\mathbf{x})$ is divided into

```
k ← 0
repeat
    // Use reduced Jacobian every qth iteration
    // after the first k = p + 1 iterations
    if ((k > p) and !(k%q)) then
        Use reduced Jacobian
    else
        Use constant Jacobian
    end if
    k ← k + 1
until Convergence
```

Difference in temperature on ignoring thermal conductivity

- Difference in temperature in a silicon layer between having a constant thermal conductivity (27° C) and incorporating nonlinearity
- The chip dimension is $8\text{mm} \times 8\text{mm}$ and it dissipates 100 W uniformly



Conclusion

- RMS response is an upper bound on the steady state response when thermal analysis is done at transistor level
- Accurate thermal analysis needs to consider nonlinear thermal conductivity
- An efficient algorithm to solve the system of nonlinear equations has been proposed