Accurate Thermal Analysis Considering Nonlinear Thermal Conductivity

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ISQED 2006
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Why thermal analysis?
- Temperature affects performance
- Temperature and power are tightly coupled
- Reliability issues

Is *Electrothermal* analysis any different in *digital* domain compared to *analog* domain?
- Essentially involves solving $A(x)x = b$
- The **size** of $A$ is the biggest challenge
Difference in temperature on ignoring thermal conductivity
A partial differential equation of the form,

$$\rho C_p \frac{\partial T(x, y, z, t)}{\partial t} = \kappa(T(x, y, z, t)) \nabla^2 T(x, y, z, t) + h(x, y, z, t)$$

where

- $\kappa(T(x, y, z, t))$ is the thermal conductivity
- $h(x, y, z, t)$ is a heat source at $(x, y, z, t)$.
- $\rho C_p$ is the heat capacity

Since we are interested in steady state, the above equation reduces to,

$$\kappa(T(x, y, z)) \nabla^2 T(x, y, z) + h(x, y, z) = 0$$
Let us consider the problem in one dimension and in one layer (silicon) to simplify things. The heat conduction equation reduces to,

\[ \kappa_{si}(T) \frac{\partial^2 T(x)}{\partial x^2} + h(x) = 0 \]

Applying finite difference to the heat conduction equation,

\[ \kappa_{si}(T_i) \left( \frac{T_{i+1} - T_i}{\Delta x} \right) - \left( \frac{T_i - T_{i-1}}{\Delta x} \right) \frac{\Delta x}{\Delta x} + h(x) = 0 \]

\[ \kappa_{si}(T_i) \left( \frac{2T_i - T_{i-1} - T_{i+1}}{(\Delta x)^2} \right) = h(x) \]

Can be written in matrix form as \( K(T)T = h \).
Background

Electrical Interpretation in 2-dimension

\[
\begin{align*}
\kappa_{si}(T_{i,j}) \Delta x & \quad \frac{\Delta y}{\Delta x} \\
\kappa_{si}(T_{i,j}) \Delta x & \quad \frac{\Delta y}{\Delta x} \\
(i, j - 1) & \quad (i, j) \\
(i, j) & \quad (i, j + 1)
\end{align*}
\]

\[
(\Delta x \Delta y) \times h_{\text{power}}
\]

\[
(i + 1, j)
\]
In literature, $K(T)T = h$ is solved as $KT = h$ [Cheng 1998, Wang 2003]

- Linear system of equations
- Still difficult to solve due to sheer size
- Consider a $8000\mu m \times 8000\mu m$ chip with grid size of $\Delta x \times \Delta y = 10\mu m \times 10\mu m$
  - The size of the matrix $K$ is $\mathbb{R}^{640,000\times640,000}$
  - Fortunately, the matrix is sparse

To get an accurate solution need to solve the nonlinear system of equations $K(T)T = h$

- Very hard since there are no black box methods to solve a nonlinear system of equations
Steady state thermal circuit

- We study if steady state (SS) heat sources can be modeled as a DC source having the RMS value of SS.
- The steady state problem is studied in $z$ direction, assume uniformity in $x, y$ directions.
- Picture of chip layers from [Su, ISLPED 2003].
Steady State Response

- The RMS analysis is done by setting the DC sources to zero and setting the ac source to the RMS value.
- The temperature rise at the substrate calculated using the RMS values for the ac sources is $2.65^\circ \text{C}$.
- The steady state response turns out to be zero!
- Thus, the approximation is good.
The thermal time constant is dominated by the substrate which is around $K_{Hz}$ and the operating frequency is $G_{Hz}$.
Nonlinear thermal conductivity

- Thermal conductivity is a nonlinear function of temperature
  - There is a 22% change in thermal conductivity of Silicon over the range of $[27, 80] \degree C$
  - Thermal gradients calculated assuming constant thermal conductivity is not very accurate
- Challenge is designing efficient algorithms to solve system of $m$ nonlinear equations simultaneously
Nonlinear thermal conductivity

Algorithms to solve nonlinear equations

- **Newton-Raphson**

\[
x^{(k)} = x^{(k-1)} - [J(x^{(k-1)})]^{-1} f(x^{(k-1)})
\]

- Simple and fast convergence
- Need to factorize the Jacobian matrix during every iteration

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**Nonlinear thermal conductivity**

**Algorithms to solve nonlinear equations**

- **Constant Jacobian**

\[ x^{(k)} = x^{(k-1)} - [J(x^{(0)})]^{-1} f(x^{(k-1)}) \]

- Only *one* factorization
- But slow convergence

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Sketch of Proposed Algorithm

- Accelerate **Constant Jacobian**

\[ f(x) \]

\[ x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_6 \]

\[ x_{roj} \rightarrow x_{cj} \rightarrow x_{cj} \rightarrow x_{cj} \rightarrow x_{cj} \rightarrow x_{cj} \]

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Proposed Algorithm

**Input:** $F(x) = A(x)x - b$: $m$ nonlinear equations in $m$ unknowns

**Input:** $p$, the number of partitions $A(x)$ is divided into

1: $k \leftarrow 0$
2: **repeat**
3: // Use reduced Jacobian every $q$th iteration
4: // after the first $k = p + 1$ iterations
5: **if** ($(k > p) \text{ and } !(k \% q))$ **then**
6: Use reduced Jacobian
7: **else**
8: Use constant Jacobian
9: **end if**
10: $k \leftarrow k + 1$
11: **until** Convergence
Difference in temperature on ignoring thermal conductivity

- Difference in temperature in a silicon layer between having a constant thermal conductivity (27°C) and incorporating nonlinearity.
- The chip dimension is 8mm × 8mm and it dissipates 100 W uniformly.

![Graph showing temperature difference](image-url)
Conclusion

- RMS response is an upper bound on the steady state response when thermal analysis is done at transistor level.
- Accurate thermal analysis needs to consider nonlinear thermal conductivity.
- An efficient algorithm to solve the system of nonlinear equations has been proposed.