L-Shape Based Layout Fracturing for E-Beam Lithography

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Abstract

Layout fracturing is a fundamental step in mask data preparation and e-beam lithography (EBL) writing. To increase EBL throughput, recently a new L-shape writing strategy is proposed, which calls for new L-shape fracturing, versus the conventional rectangular fracturing. Meanwhile, during layout fracturing, one must minimize very small/narrow features, also called slivers, due to manufacturability concern. This paper addresses this new research problem of how to perform L-shaped fracturing with sliver minimization. We propose two novel algorithms. The first one, rectangular merging (RM), starts from a set of rectangular fractures and merges them optimally to form L-shape fracturing. The second algorithm, direct L-shape fracturing (DLF), directly and effectively fractures the input layouts into L-shapes with sliver minimization. The experimental results show that our algorithms are very effective.

1. Introduction

E-Beam lithography (EBL) [1] is widely deployed in the mask manufacturing, which is a significant step affecting the fidelity of the printed image on the wafer and critical dimension (CD) control. Because of the capability of accurate pattern generation, EBL is also a promising candidates for sub-22nm logic nodes, along with extreme ultra violet (EUV) [2] and double/multiple patterning lithography (DPL/MPL) [3][4]. For EBL writing, a fundamental step is *layout fracturing*, where the layout pattern is decomposed into numerous non-overlapping rectangles. Subsequently the layout is prepared and exposed by an EBL writing machine onto the mask or the wafer, where each fractured rectangle is shot by one variable shaped beam (VSB).

As the minimum feature size further decreases, the number of rectangles in the layout is steadily increased. First, longer writing time and larger data volume are caused by highly complex optical proximity correction (OPC). Besides, the introduction of advanced lithographic techniques, e.g., DPL/MPL, add more masks in the mask manufacturing. Since the manufacturing cost is directly associated with increasing write time and data volume, the cost is also steadily increased. In addition, the low throughput has been and is still the bottleneck of EBL writing.

To overcome this manufacturing problem, several optimization methods have been proposed to reduce the EBL writing time to a reasonable level [5][6][7]. Among them, the L-shape shot strategy is a very simple yet effective approach to reduce the e-beam mask writing time, and thus reduce the mask manufacturing cost and improve the throughput [5][6]. Besides, this technique can be also applied to reduce the cost of lithographic process. The conventional EBL writing is based on rectangular VSB shots. As illustrated in Fig. 1(a), the electrical gun generates an initial beam, which becomes uniform through the shaping aperture. Then the second aperture finalizes the target shape with a limited maximum size. As an improved technique, the printing process of the L-shape shot is illustrated in Fig. 1(b). One additional aperture, the third aperture, is employed to create L-shape shots. To take advantage of this new printing process, new fracturing methodology is needed to provide L-shape in the fractured layout. L-shape shot strategy has the potentiality to reduce the EBL writing time or cost by 50% if
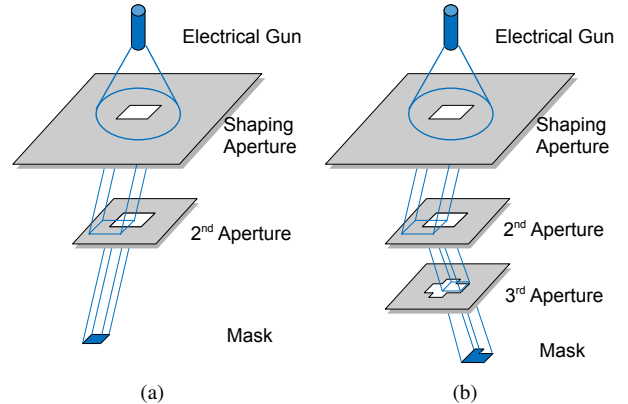

Figure 1: (a) Traditional rectangular EBL writing process. (b) L-shape writing process with one additional aperture.
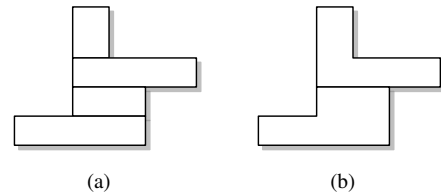

Figure 2: Examples of polygon fracturing. (a) Rectangular shots with 4 shot number. (b) L-shape shots with 2 shot number.
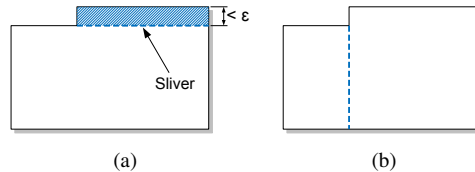

Figure 3: (a) Fracturing with one sliver. (b) Fracturing without sliver.

all rectangles are combined into L-shapes. For example in Fig. 2, instead of four rectangles, using L-shape fracturing only requires two L-shape shots.

Note that the layout fracturing problem is different from the general polygon decomposition problem in geometrical science. In order to consider yield control and CD control, the minimum width of each shot should be above a certain threshold value ϵ. A shot whose minimum width is $< \epsilon$ is called a *sliver*. In the layout fracturing, sliver minimization is an important objective [8]. As shown in Fig. 3, two fractured layouts can achieve the same shot number 2. However, because of sliver, the fractured result in Fig. 3 (a) is worse than that in Fig. 3 (b). It shall be noted that the layout in Fig. 3 can be written in one L-shaped shot without any sliver.

For traditional rectangular shots, several papers have studied the layout fracturing problem [8][9][10][11][12]. Kahng et. al proposed an integer linear programming (ILP) formulation, and some

matching based speed-up techniques [8][9]. Recently, Ma et. al [11] presented a heuristic algorithm to generate rectangular shots and further reduce the sliver. Compared with the rectangular fracturing problem, the L-shape fracturing problem is new and there is only limited work, mostly describing methodology, but no systematic algorithm has been proposed so far. [5] reported the initial results that L-shape fracturing can further save about 38% of shot count, but no algorithmic details are provided. For the general decomposition problem of polygon into L-shapes, several heuristic methods are proposed [13][14]. However, since these heuristic methods only consider horizontal decomposition, which would result in numerous slivers, they cannot be applied to the layout fracturing problem.

This paper presents the first systematic study for EBL L-shape fracturing considering the sliver minimization. We propose two algorithms for the L-shape fracturing problem. The first method, called *RM*, starts from rectangles generated by any previous fracturing framework, and merge them into L-shapes. A maximum weighted matching algorithm is proposed to find the optimal merging solution, where the shot count and the sliver can be minimized simultaneously. To further overcome the intrinsic limitations of rectangular merging, we propose another fracturing algorithm, called *DLF*. Through effectively detect and take advantage of some special cuts, DLF can directly fracture the layout into a set of L-shapes in $O(n^2 log n)$ time. The experimental results show that our algorithms are very promising for both shot count reduction and sliver minimization. In addition, DLF can even achieve significant speed-up compared with previous state-of-the-art rectangular fracturing algorithm [11].

The rest of the paper is organized as follows. Section 2 presents the basics and problem formulation. Section 3 provides RM, the merging based algorithm, which will also be used as a baseline. In Section 4 we propose the DLF algorithm to directly fracture polygons into L-shapes. Section 5 presents experimental results, followed by conclusion in Section 6.

2. Definitions and Problem Formulation

We first introduce some notations and definitions to facilitate the problem formulation. For convenience, we use the term polygon to refer to rectilinear polygons in the rest of this paper.

Let P be an input polygon with n vertices, we define the concave vertices as follows.

Definition 1 (Concave Vertex). *The concave vertex of a polygon is one at which the internal angle is 270°.*

Let c be the number of concave vertices in P, [15] gave the relationship between n and c: $n = 2c+4$. If the number of concave vertices c is odd, polygon P is called *odd polygon*; otherwise, P is called *even polygon*.

Definition 2 (Cut). *A cut of a polygon P is a horizontal or vertical line segment at least one of whose endpoints is incident on a concave vertex. The other endpoint is obtained by extending the line segment inside P until it first encounters the boundary of P.*

If both endpoints of a cut are concave vertices in the original polygon, then the cut is called a *chord*. If a cut has odd number of concave vertices to one side or another, then the cut is called an *odd-cut*. If an cut is not only odd-cut but also chord, it is called an *odd-chord*. These concepts are illustrated in Fig. 4, where vertices b, e, h are concave vertices, edges $\bar{bh}, \bar{ej}$ are odd-cuts, and edge $\bar{bh}$ is chord. Note that $\bar{bh}$ is an odd-chord.

Definition 3 (L-shape). *An L-shape is a polygon shaped in the form of the letter L.*

An L-shape can be also viewed as a combination of two rectangles with a common coordinate. There are two easy ways to check
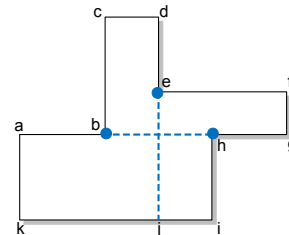

Figure 4: Concepts of concave vertices and cuts.

whether a polygon is an L-shape. First, we can check whether the number of vertices equals to 6, i.e., $n = 6$. Besides, we can check whether there is only one concave vertex, i.e., $c = 1$.

Definition 4 (Sliver Length). *For an L-shape or a rectangle, if the width of its bounding box B is above ϵ, its sliver length is 0. Otherwise, the sliver length is the length of B.*

Problem 1 (L-shape based Layout Fracturing). *Given an input layout which is specified by polygons, our goal is to fracture it into a set of L-shapes and/or rectangles to minimize the number of shots, and meanwhile minimize the silver length of fractured shots.*

3. Rectangular Merging (RM) Algorithm

Given the rectangles generated by any rectangular fracturing methodology, we propose an algorithm, called RM, to merge them into a set of L-shapes. The main idea is that if two rectangles share a common coordinate, they can be combined into one L-shape. Although this idea is straightforward, the benefit is obvious that the previous rectangular fracturing algorithms can be re-used. Besides, the RM algorithm is used as a baseline in comparison with our another algorithm, DLF, which will be described in Section 4.
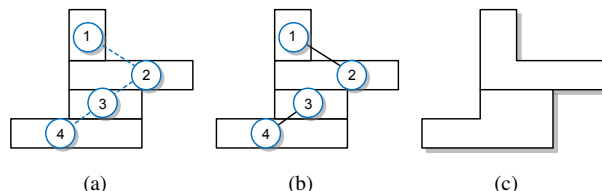

Figure 5: Example of RM algorithm. (a) Graph construction. (b) Maximum matching result. (c) Corresponding rectangular merging.

Given the input rectangles, the RM algorithm can find the optimal L-shape merging solution. Meanwhile, the shot count and sliver length can be minimized simultaneously.

First we construct a merging graph G to represent the relationships among all the input rectangles. Each vertex in G represents a rectangle. There is an edge between two vertices if and only if those two rectangles can be merged into an L-shape. For example shown in Fig. 5, after rectangular fracturing, four rectangles are generated. The constructed merging graph G is illustrated in Fig. 5(a), where the three edges show that there are three ways to generate L-shapes. L-shape merging can be viewed as edge selection from the merging graph G. Note that one rectangle can only be assigned to one selected edge, that is, no two selected edges share a common end point. For example, rectangle 2 can only belongs to one L-shape, and thus only one edge can be chosen between edges $\bar{12}$ and $\bar{23}$.

By utilizing the merging graph, the best edge selection can be solved by finding a maximum matching. Therefore, the rectangular merging can be formulated as a maximum matching problem. In the case of Fig. 5, the result of the maximum matching is illustrated in Fig. 5(b), and the corresponding L-shape fracturing result is shown in Fig. 5(c).

In order to consider the sliver minimization, we assign weights to the edges to represent whether the merging would remove one sliver. For example, if there is still one sliver even two rectangles v_i and v_j are merged into one L-shape, we assign less weight to edge e_{ij}. Otherwise, larger weight is assigned. Therefore, the rectangular merging can be formulated as maximum weighted matching. Even in general graphs, the maximum weighted matching can be solved in $O(nm\log n)$ time [16], where the n is the number of vertices, and the m is the number of edges in G.

4. Direct L-Shape Fracturing (DLF) Algorithm

Although the RM algorithm described above can provide the optimal merging solution for given rectangles, it may suffer from several limitations. First, the polygon is fractured into rectangles first, and followed by a merging stage. This strategy, however, has some redundant or unnecessary operations. For the case in Fig. 2, instead of complex rectangles generation, only one cut is enough for the L-shape fracturing. Second, the rectangular fracturing may ignore some internal features of L-shape fracturing, which could sacrifices the whole performance. To overcome all these limitations, in this section we propose a novel algorithm, called DLF, to directly fracture polygon into L-shapes.

We observe that the solution space for the L-shape fracturing can be very large. Given a polygon, there can exist several fracturing solutions with the same shot count. For example, as shown in Fig. 6, the input polygon has at least five different fracturing solutions with two shots.
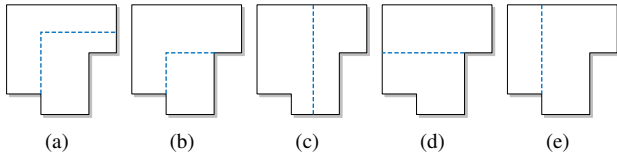

(a)   (b)   (c)   (d)   (e)

Figure 6: Five fracturing solutions with the same shot count.

Note that a cut has the following property: if the polygon is decomposed through a cut, the concave vertex that is one of the endpoints of the cut is no longer concave in either of the two resulting polygons. Our L-shape fracturing algorithm, DLF, takes advantage of this property. Each time a polygon is cut, DLF searches one appropriate odd-cut to decompose the polygon. It was shown in [15] that odd-cut always exists and $\lfloor c/2 \rfloor + 1$ "guards" are necessary and sufficient to cover all the interiors of a polygon with c concave vertices. Therefore, we can obtain the following lemma.

Lemma 1. *A polygon with c concave vertices can be decomposed into L-shapes with upper bound number $N_{up} = \lfloor c/2 \rfloor + 1$.*
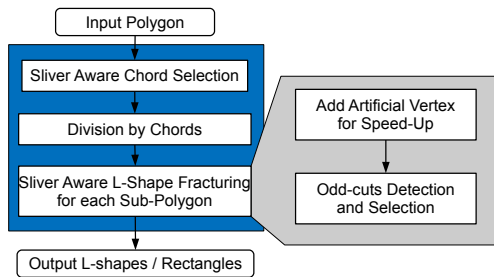

Figure 7: Overall flow of DLF algorithm.

Fig. 7 shows the overall flow of our DLF algorithm. We will effectively use chords and cuts to reduce the problem size while containing or even reducing the L-shape fracturing number upper bound. The first step is to detect all chords (i.e., horizontal or vertical cuts made by concave points), in particular odd-chords as they

may reduce the L-shape upper bound. We will then perform sliver-aware chord selection to decompose the original polygon P into a set of sub-polygons. Then for each sub-polygon, we will perform sliver aware L-shape fracturing, where odd-cuts are detected and selected to iteratively cut the polygon into a set of L-shapes. The reason we differ chord and cut during polygon fracturing is that chord is a special cut with both end points being concave points in the original polygon. That way, we can design more efficient algorithm for odd cut/chord detection.

4.1 Sliver Aware Chord Selection

The first step of DLF algorithm is sliver aware chord selection. Cutting through chords decomposes the whole polygon P into a set of sub-polygons. By this way the problem size is reduced. We can further prove that cutting through a chord does not increase the L-shape upper bound N_{up}.

Lemma 2. *Decompose a polygon through a chord does not increase the L-shape upper bound number N_{up}.*

Proof. Cut the polygon along this chord, and let c_1 and c_2 be the number of concave vertices in the two pieces produced. Since $c = c_1 + c_2 + 2$, then using Lemma 1 we have

$$\lfloor c_1/2 \rfloor + 1 + \lfloor c_2/2 \rfloor + 1 \leq \lfloor (c_1 + c_2)/2 \rfloor + 2$$
$$= \lfloor (c-2)/2 \rfloor + 2 = \lfloor c/2 \rfloor + 1$$

□

Chord selection has been proposed in rectangular fracturing [9][11], but for L-shape fracturing, odd-chord shall be selected as they can even reduce the number of L-shapes.

Lemma 3. *Decomposing a even polygon along an odd-chord can reduce the L-shape upper bound number N_{up} by 1.*

The proof is similar to that for Lemma 2. The only difference is that since c is even and c_1, c_2 are odd, $\lfloor c_1/2 \rfloor + 1 + \lfloor c_2/2 \rfloor + 1 < \lfloor c/2 \rfloor + 1$. Note that for an odd polygon, all chords are odd. For a even polygon, Lemma 3 provides a guideline to select chords. An example is illustrated in Fig. 8, which contains two chords $\bar{b}\bar{h}$ and $\bar{h}\bar{k}$. Since the number of concave vertices to both side of chord $\bar{b}\bar{h}$ are odd (1), $\bar{b}\bar{h}$ is an odd-chord. Cut along $\bar{b}\bar{h}$, as shown in Fig. 8(a), can achieve two L-shots. However, cut along another chord $\bar{h}\bar{k}$, which is not an odd-chord, would need three shots. Note that in an odd polygon, although all chords are odd, cutting along them may not reduce N_{up}, but it will not increase N_{up} either.
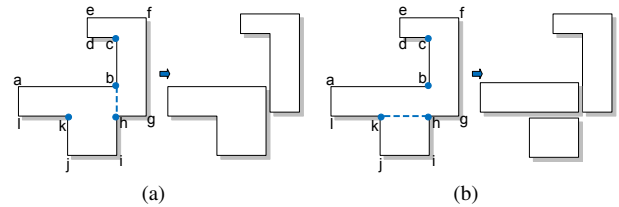

(a)   (b)

Figure 8: Examples to illustrate Lemma 3. (a) Cut along odd-chord $\bar{b}\bar{h}$ results in two L-shape shots. (b) Cut along chord $\bar{h}\bar{k}$ would cause one more shot.

For any even polygon P, we propose the odd-chord search procedure as follows. Each vertex v_i is assigned with one Boolean parity p_i. Starting from an arbitrary vertex with any parity assignment, we proceed clockwise around the polygon. If the next vertex v_j is concave, then $p_j = \neg p_i$, where p_i is the parity of current vertex v_i. Otherwise p_j is assigned to p_i. This parity assignment can be completed during one clockwise traverse in $O(n)$ time. An example of parity assignment starting from a(0) is shown in Fig. 9, where each vertex is associated with one parity.
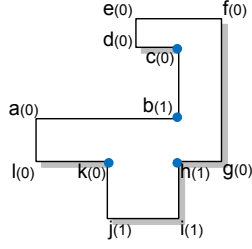
Figure 9: To detect odd-chords in even polygon, each vertex is associated with one Boolean parity.

Theorem 1. *In a even polygon, a chord $\bar{ab}$ is odd iff $p_a = p_b$.*

Given the parity values, the odd-chord detection can be performed using Theorem 1. For each concave vertex v_i, a plane sweep is applied to search any chord containing v_i. The plane sweep for each vertex can be finished in $O(logn)$, and the number of vertices is $O(n)$. Therefore, in an even polygon odd-chords detection can be completed in $O(nlogn)$ time.

After all chords are detected, chord selection is applied to choose as many chords as possible to divide the input polygon into a set of independent sub-polygons. Note that if a chord is selected to cut the polygon, it releases the concavity of its two endpoints. Therefore, if two chords intersect with each other, at most one of them could be selected. For example, in Fig. 9, chords $\bar{bh}$ and $\bar{hk}$ cannot be selected simultaneously. The relationship among the chords can be represented as a bipartite graph [9], and the vertices in left and right columns indicate the horizontal and vertical chords, respectively. Therefore, finding the most chords compatible with each other corresponds to finding the maximum independent set in the bipartite graph, which can be reduced to maximum matching problem, and therefore, can be done in polynomial time. It shall be noted that if the input polygon is a even polygon, because of Theorem 1, we prefer to choose odd-chords. Therefore, the bipartite graph is modified by assigning weights to the edges. In addition, sliver minimization is integrated into the chord selection. When an odd-chord candidate is detected, we calculate the distance between it and the boundary of the polygon. If the distance is less than ϵ, cutting this odd-chord would cause sliver, then we will discard this candidate.

4.2 Sliver Aware L-Shape Fracturing

After chord selection, the input polygon P is decomposed into m sub-polygons (denoted as $P_1, P_2, ..., P_m$). For each polygon P_i, we will recursively fracture it until reaching final L-shapes and/or rectangles. Our fracturing algorithm is based on odd-cut selection. The main idea is that each time we pick up one odd-cut, and decompose the polygon into two pieces through this odd-cut. Iteratively we fracture the polygon into several L-shapes. Note that our fracturing algorithm considers the sliver minimization, i.e., we try to minimize the sliver length during fracturing.

The first question is how to detect all the odd-cuts efficiently. Our method is similar to that for odd-chord detection. Each vertex v_i is assigned an order number o_i, and a Boolean parity p_i. Start at an arbitrary vertex, each vertex v_i is assigned an order o_i. We initialize the Boolean parity p to zero, and proceed clockwise around the polygon. If the next vertex v_i is normal, label its p_i as p; if v_i is concave, assign p to $\neg p$, and label its p_i with the new p value. For each concave vertex v_a, we search cuts from two directions (horizontal and vertical) from it. Here we denote $(a, \bar{bc})$ as the cut with one endpoint at vertex v_a and the other endpoint at edge $\bar{bc}$. For each cut $(a, \bar{bc})$ detected, whether it is an odd-cut can be checked in constant time using the following Theorem 2.

Theorem 2. *In an odd polygon, a cut $(a, \bar{bc})$ is an odd-cut if and only if the following condition is satisfied:*

$$\begin{cases} p_a = p_b, & \text{if } o_a > o_b \\ p_a \neq p_b, & \text{if } o_a < o_b \end{cases}$$

Due to space limit, the detailed proof is omitted. An example of odd-cut detection is shown in Fig. 10. There are three concave vertices, v_b, v_f and v_i in the odd polygon. Start from each concave vertex, we have searched all six cuts. Applying Theorem 2, we find out two odd-cuts $(b, \bar{fg})$ and $(i, \bar{cd})$.
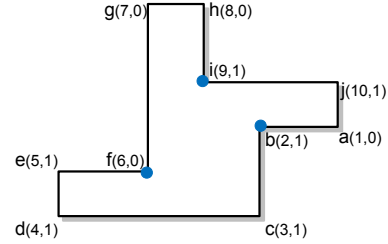

Figure 10: Odd-cut detection using order number and parity.

Algorithm 1 LShapeFracturing(P)

Require: Polygon P.
1: **if** P is L-shape or rectangle **then**
2:     Output P as one of results;
3:     **return**
4: **end if**
5: Find all odd-cuts;
6: Choose cut cc considering the sliver minimization;
7: **if** Cannot find legal odd-cut **then**
8:     Generate an auxiliary cut cc;
9: **end if**
10: Cut P through cc into two polygons $P1$ and $P2$;
11: Update one vertex and four edges;
12: LShapeFracturing($P1$);
13: LShapeFracturing($P2$);

The details of our L-shape fracturing are described in Algorithm 1. Given the input polygon P, if it is already an L-shape or rectangle, then the fracturing is completed. Otherwise, we find all odd-cuts as described above (line 5). From all the odd-cuts detected, we choose one cc, and cut the P into two pieces P_1 and P_2 (lines $10 - 11$). Then we recursively apply L-shape fracturing to P_1 and $P2$ (lines $12 - 13$).

Note that during the polygon decomposition, we do not need to re-calculate the order number and parity of each vertex. Instead, when a polygon is divided into two parts, we only update one vertex and four edges, while all other information can be maintained. If polygon P is cut through odd-cut $(a, \bar{bc})$, a new vertex, namely d, is generated. For the new vertex d, its order number $o_d = o_b$ and its parity $p_d = p_b$. Edge $\bar{bc}$ is replaced by edges $\bar{bd}$ and $\bar{dc}$. Besides, two edges $\bar{ad}$ and $\bar{da}$ are inserted. The update method is simple and easy to implement. An example of such update is shown in Fig. 11.

Sliver minimization is integrated into our L-shape fracturing algorithm. In Algorithm 1, when picking up one cut from all odd-cuts, we try to avoid any sliver. For example, as illustrated in Fig. 12(a), there are three odd-cuts, but all of them would cause sliver. Instead of selecting any of them, we generate an auxiliary cut in the middle (see Fig. 12(b)). Because of the auxiliary cut, the polygon can be fractured without introducing any sliver. In addition, if there are several odd-cuts not causing sliver, we pick the cut using the following rules: (1) We prefer the cut which partitions
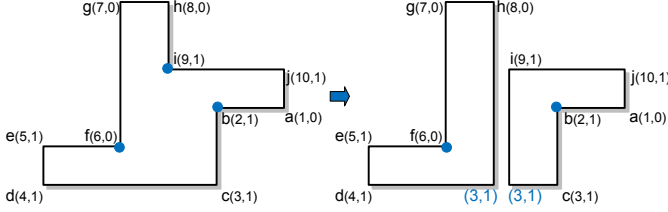
Figure 11: Only one vertex and four edges need to be updated during polygon decomposition.

the polygon into two balanced sub-polygons; (2) If the polygon is more horizontal than vertical, we prefer a vertical cut, and vice versa.
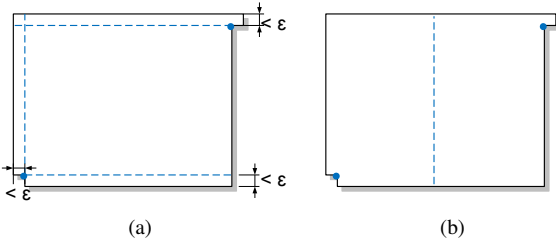

(a)   (b)

Figure 12: Auxiliary cut generation. (a) Here every odd-cut would cause sliver. (b) Decompose through on auxiliary cut can avoid sliver.

Given a polygon with n vertices, finding all concave vertices need $O(n)$ time. For each concave vertex v_i, searching cut starting from it needs $O(logn)$ time. Using Theorem 2, checking whether the cut is odd-cut needs $O(1)$, thus finding all odd-cuts needs $O(nlogn)$ time. Note that given a polygon with c concave vertices, if no auxiliary cut is generated, the L-shape fracturing can be completed through $\lfloor c/2 \rfloor$ odd-cuts. When auxiliary cuts are applied, there are at most $c - 1$ cuts to fracture the input polygon. Therefore, we can achieve the following theorem.

Theorem 3. *The sliver aware L-shape generation can find a set of L-shapes in $O(n^2 logn)$ time.*

It shall be noted that if our objective is only to minimize the shot number, no auxiliary cut would be introduced, thus at most $\lfloor c/2 \rfloor + 1$ L-shapes are generated. In other words, the shot number would be less or equals to the theoretical upper bound N_{up}.

4.3 Speedup Technique

We observe that in practice during the execution of Algorithm 1, many odd-cuts do not intersect. In other words, many odd-cuts are compatible, and could be used to decompose the polygon at the same time. Instead of only picking one odd-cut at one time, we can achieve further speed-up by selecting multiple odd-cuts simultaneously.

If the polygon is an odd polygon, this speed-up is easily implemented. In the odd polygon, there is only one type of odd-cut: a cut that has an odd number of concave vertices to each side. Partitioning the polygon along such odd-cut can leave all other odd-cuts remaining to be odd-cuts. For example, Fig. 13(a) shows an odd polygon, where all three odd-cuts are compatible, and can be picked up simultaneously. Through fracturing the polygon along the three odd-cuts, the L-shape fracturing problem is resolved directly.

However, this speed-up technique cannot be directly applied to an even polygon, since it may cause more shot number. The reason is that when an even polygon is cut into two pieces, some odd-cuts may no longer be odd-cuts in the sub-polygons. For example, as shown in Fig. 14(a), in this even-polygon all six cuts are odd-cuts and compatible. However, if we use all these compatible cuts for
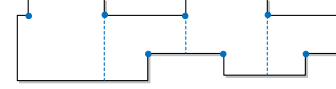

Figure 13: Speed-up for odd polygon, where all three odd-cuts are compatible.

fracturing, we would end up with seven rectangular shots, which is obviously sub-optimal. To overcome this issue, for each even-polygon we introduce one artificial concave vertex. Through this artificial concave vertex, the polygon is translated into an odd polygon. Because of Lemma 4, this translation does not increase the total shot number. As shown in Fig. 14(b), in the modified odd polygon, all compatible odd-cuts can be used for fracturing without causing more shot number.

Lemma 4. *Introducing one artificial concave vertex to an even polygon does not increase the L-shape upper bound N_{up}.*
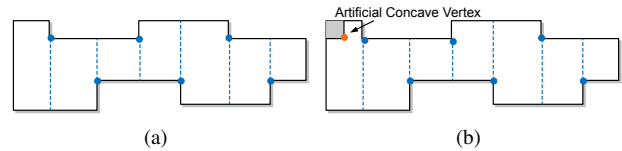

(a)   (b)

Figure 14: Speed-up for even polygon. (a) all cuts are odd-cuts. (b) Introducing one artificial concave vertex, translate the even polygon into an odd polygon.

Through employing this speed-up technique, for most cases, the odd-cut detection can be applied only once, therefore the DLF algorithm could be completed in $O(nlogn)$ time in practice.

5. Experimental Results

We implemented our two L-shape fracturing algorithms, RM and DLF, in C++. Since RM needs a rectangular fracturing method to generate initial rectangles, we implemented a state-of-the-art algorithm proposed in [11]. Based on the generated rectangles, RM algorithm is applied to merge them into a set of L-shapes. LEDA package [17] is adopted for the maximum weighted matching algorithm.

The experiments are performed on an Intel Xeon 3.0GHz Linux machine with 32G RAM. ISCAS 85&89 benchmarks are scaled down to 28nm logic node, followed by accurate lithographic simulations performed to the Metal 1 layers. All involved lithography simulations in the *Calibration Phase* are applied under industry-strength RET (OPC). For all the resulting post-OPC layers, OpenAccess 2.2 [18] is adopted for interfacing.

Table 1 shows the results of our DLF algorithm in comparison with the approaches in [11] and the RM algorithm. Since the framework [11] is adopted to provide the input rectangles, the RM algorithm is denoted as "[11]+RM". Column "poly#" lists the number of polygons of each test circuit. All fracturing methods are evaluated with the sliver parameter $\epsilon = 5nm$. For each method, columns "shots", "sliver", and "CPU" denote the shot number, total sliver length, and runtime, respectively. First we compare the fracturing algorithm in [11] and the RM algorithm. From the table we can see that as an incremental algorithm, the RM algorithm can further reduce the shot number by 37%, and the sliver length by 45%. Meanwhile, the runtime increasing is reasonable: RM algorithm introduces 41% more runtime. Besides, we compare our DLF algorithm with other two methods. We can see that DLF demonstrates the best performance, in terms of both runtime and performance. Compared with traditional sliver aware rectangular fracturing [11], it can achieve around $9\times$ speed-up. Besides, the shot number and the sliver length can be significantly reduced (39% and 82%, respectively). Even compared with RM algorithm, DLF is better in

Table 1: Runtime and Performance Comparisons

Circuts	poly#	[11]			[11]+RM			DLF		
		shots	sliver (μm)	CPU(s)	shots	sliver (μm)	CPU(s)	shots	sliver (μm)	CPU(s)
C432	1109	6898	48.3	8.51	4371	23.5	10.0	4214	7.4	1.87
C499	2216	13397	96.0	16.9	8325	45.0	19.5	8112	11.8	2.6
C880	2411	17586	160.5	24.93	11020	84.4	29.7	10653	28.6	3.8
C1355	3262	23283	185.2	29.44	14555	87.8	33.6	13936	24.8	5.1
C1908	5125	35657	333.6	48.78	22352	181.1	57.3	21540	88.0	7.68
C2670	7933	56619	525.4	84.11	35424	274.4	96.9	34102	114.8	11.89
C3540	10189	74632	668.5	114.33	46617	360.0	133.7	44901	129.8	15.98
C5315	14603	108761	950.4	176.89	67795	488.9	200.8	65222	190.2	23.85
C6288	14575	103148	819.2	175.65	64987	382.0	201.0	62416	86.1	22.64
C7552	21253	151643	1334.6	242.77	94902	717.6	280.0	91157	290.7	32.02
S1488	4611	37126	303.7	55.03	22984	146.2	64.7	22099	31.6	8.14
S38417	67696	454307	4040.2	727.1	285049	2293.0	1020	275054	729	88.5
S35932	26267	163956	1470.4	228.02	103960	808.3	256.4	100629	284	34.85
S38584	168319	1096363	10045.2	2268.6	690054	5777.0	3565.2	666906	1801.7	216.39
S15850	34660	231681	2012.8	329.99	145745	1085.1	414.6	140879	320	44.7
avg.	-	171670	1533.0	302.1	107876	850.3	425.6	104121	275.9	34.7
ratio	-	1	1	1	0.63	0.55	1.41	**0.61**	**0.18**	**0.11**

terms of performance: it can further reduce the shot number and the sliver length by 3.2% and 67%, respectively.
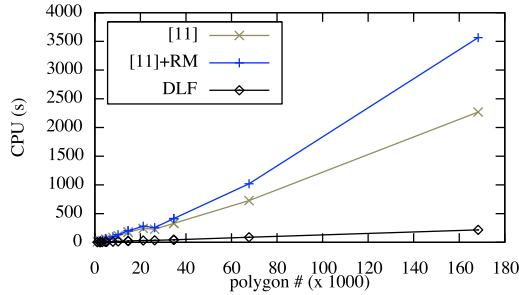

Figure 15: Comparison on algorithm scalability.

In order to evaluate the scalability of our algorithm, we summarize all the run time from Table 1, and display in Fig. 15. Here the X axis denotes the number of polygons (e.g., the problem size), and the Y axis shows the runtime. We can see that DLF algorithm scales better than both [11] and RM algorithm.

6. Conclusions

In this paper we have proposed two novel algorithms for EBL with the new L-shape based layout fracturing for shot number and sliver minimization. The rectangular merging (RM) based algorithm is optimal for a given set of rectangular fractures. However, to get better performance, we show that the direct L-shape fracturing (DLF) algorithm is superior by directly decomposing the original layouts into a set of L-shapes. DLF obtained the best results in all metrics, including shot count, sliver, as well as runtime compared to the previous state-of-the-art rectangular fracturing with RM. To our best knowledge, this is the first systematic and algorithmic effort in EBL L-shaped fracturing with sliver minimization. As EBL is widely used for mask making and also gaining momentum for direct wafer writing, we believe a lot more research can be done, for not only layout fracturing but also EBL-aware physical design.

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