6. Logical Effort

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Review: See Additional Notes Posted

Calculate the Elmore delay from C to F in the circuit. The widths of the pass transistors are shown, and the inverters have minimum-sized

Use the Elmore delay approximation to find the *worst-case* rise and fall delays at output F for the following circuit. The gate sizes of the transistors are shown in the figure. Assume NO sharing of diffusion regions, and the worst-case conditions for the initial charge on a node.

Example: Delay with Different Input Sequences

Find the delays for the given input transitions (gate sizes shown in figure)

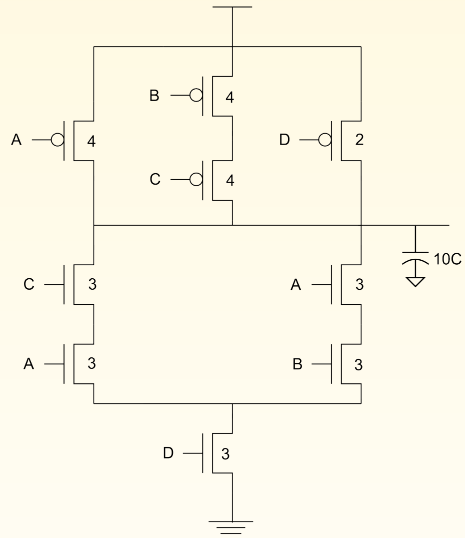
Assumptions: diffusion capacitance is equal to the gate capacitance, the resistance of an nMOS transistor with unit width is R and the resistance of a pMOS transistor with width 2 is also R, and NO sharing of diffusion regions

Off-path capacitances can contribute to delay, and if a node does not need to be charged (or discharged), its capacitance can be ignored

$$ABCD = 0101 \rightarrow ABCD = 1101$$

$$ABCD = 1111 \rightarrow ABCD = 0111$$

$$ABCD = 1010 \rightarrow ABCD = 1101$$



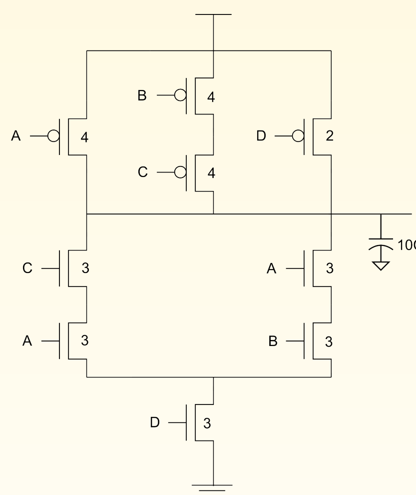
Delay with Different Input Sequence, Cont'd

Look at the charges on the nodes at the end of the first input of the sequence; only the capacitances of the nodes which would change with the second vector need to be considered

$ABCD = 0101 \rightarrow$
$ABCD = 1101;$
Delay = 36RC

$ABCD = 1111 \rightarrow$
$ABCD = 0111;$
Delay = 16RC

$ABCD = 1010 \rightarrow$
$ABCD = 1101;$
Delay = 43RC



Delay Components

Delay has two parts

Parasitic Delay

- 6 or 7 RC
- Independent of Load

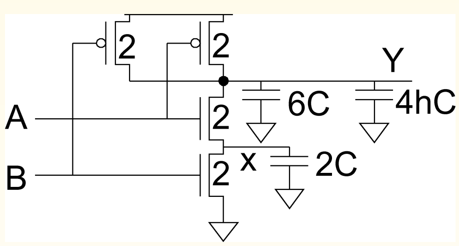
Effort Delay

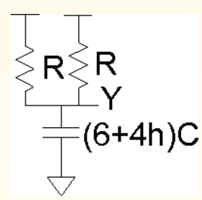
- 4h RC
- Proportional to load capacitance

Contamination Delay

Minimum (Contamination) Delay

- Best-case (contamination) delay can be substantially less than propagation delay
- Example, If both inputs fall simultaneously
- Important for "hold time" (will see later in the course)





$$t_{cdr} = (3 + 2h)RC$$

Introduction to Logical Effort

Chip designers have to face a bewildering array of choices

- What is the best circuit topology for a function?
- How many stages of logic give least delay?
- How wide should the transistors be?

Logical effort is one method to make these decisions

- Uses a simple model of delay
- Allows back-of-the-envelope calculations
- Helps make rapid comparisons between alternatives
- Emphasizes remarkable symmetries

Example

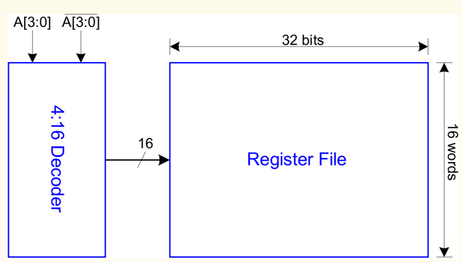
Design the decoder for a register file

Decoder specifications

- 16 word register file
- Each word is 32 bits wide
- Each bit presents load of 3 unit-sized transistors
- True and complementary address inputs A[3:0]
- Each input may drive 10 unit-sized transistors

Need to decide

- How many stages to use?
- How large should each gate be?
- How fast can decoder operate?



Delay in a Logic Gate

Express delay in a process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

$\tau = 3RC$

$\approx$ 12 ps in 180 nm process

40 ps in 0.6 μm process

Delay has two components: d = f + p

Effort delay, f = gh (stage effort)

g: Logical Effort

Measures relative ability of gate to deliver current

g $\equiv$ 1 for inverter

h: Electrical Effort=C_{out}/C_{in}

Ratio of output to input capacitances, sometimes called fanout effort

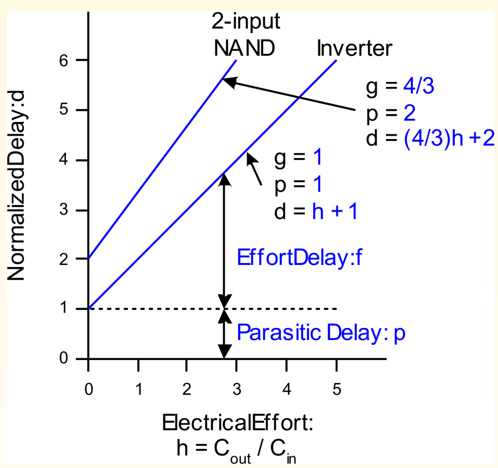
Parasitic delay, p

- Represents delay of gate driving no load
- Set by internal parasitic capacitance

Delay Plots

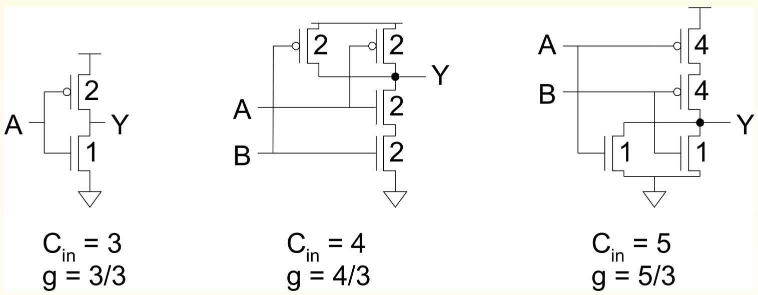
$$d = f + p$$
$$= gh + p$$

What about NOR2?



Computing Logical Effort

- **Logical effort is the ratio of the input capacitance of a gate to the input capacitance of an inverter delivering the same output current**
- Measure from delay vs. fanout plots
- Or, estimate by counting transistor widths



Catalog of Gates

- **Logical Effort of common gates**

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		4/3	5/3	6/3	(n+2)/3
NOR		5/3	7/3	9/3	(2n+1)/3
Tristate/Mux	2	2	2	2	2
XOR, XNOR		4,4	6,12,6	8,16,16,8	

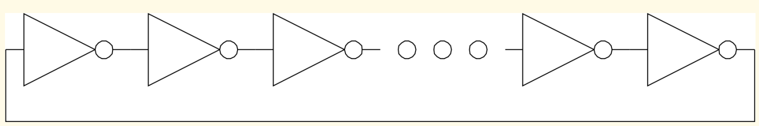
Catalog of Gates

- **Parasitic delay of common gates**
  - In multiples of $p_{inv} (\approx 1)$

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		2	3	4	n
NOR		2	3	4	n
Tristate/Mux	2	4	6	8	2n
XOR, XNOR		4	6	8	

Example: Ring Oscillator

Estimate the frequency of an N-stage ring oscillator



Logical Effort: g = 1

Electrical Effort: h = 1

Parasitic Delay: p = 1

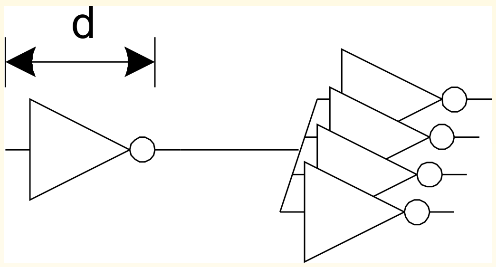
Stage Delay: d = 2

Frequency: $f_{osc} = 1/(2 \cdot N \cdot d) = 1/4N$

31 stage ring oscillator in 0.6 μm process has frequency of $\sim$ 200 MHz

Example: FO4 Inverter

Estimate the delay of a fanout-of-4 (FO4) inverter



Logical Effort: g = 1
Electrical Effort: h = 4
Parasitic Delay: p = 1
Stage Delay: d = 5

The FO4 delay is about:
200 ps in a 0.6μm process
60 ps in a 180 nm process
f/3 ns in a f μm process
(f/3 ps in a f nm process)

Example Problem

A particular technology node has a FO4 delay of 9 ps. How many minimum size (2:1) inverters need to be included in a ring oscillator so that the frequency is close to 7.3 GHz?

FO4 delay $= 15RC = 9ps$
Stage delay $= 6RC = 3.6ps$

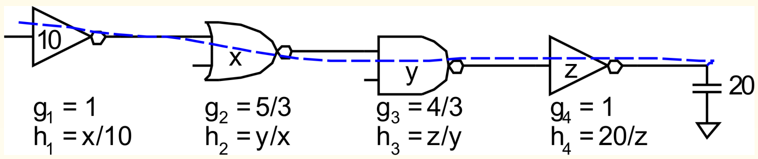
$$f = \frac{1}{2 \times N \times d} \implies$$

$$N = \frac{1}{2 \times f \times d}$$
$$= \frac{1}{2 \times 7.3 \times 3.6 \times 10^{-3}}$$
$$= 19.05$$

Number of inverters = 19

Multistage Logic Networks

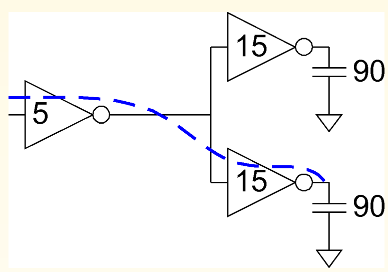
- Logical effort generalizes to multistage networks
- **Path Logical Effort**, $G = \prod g_i$
- **Path Electrical Effort**, $H = \frac{C_{out-path}}{C_{in-path}}$
- **Path Effort**, $F = \prod f_i = \prod g_i h_i$



- Can we write F = GH in general?

Consider Paths that Branch

G = 1
H = 90 / 5 = 18
GH = 18
h1 = (15 + 15)/5 = 6
h2 = 90/15 = 6
F = $g_1 g_2 h_1 h_2$ = 36 = 2GH



Branching Effort and Multistage Delays

Branching Effort accounts for branching between stages in path

$$b = \frac{C_{on\ path} + C_{off\ path}}{C_{on\ path}}$$

$$B = \prod b_i \qquad (\text{Note}: \prod h_i = BH)$$

$$\text{Now, path effort, } F = GBH$$

Multistage Delays

$$\text{Path Effort Delay, } D_F = \sum f_i$$

$$\text{Path Parasitic Delay, } P = \sum p_i$$

$$\text{Path Delay, } D = \sum d_i = D_F + P$$

Designing Fast Circuits

$$D = \sum d_i = D_F + P$$

Delay is smallest when each stage bears same effort

$$\hat{f} = g_i h_i = F^{\frac{1}{N}}$$

Thus, the minimum delay of an N-stage path is

$$\boxed{D = NF^{\frac{1}{N}} + P}$$

- This is a **key** result of logical effort
  - Find fastest possible delay
  - Doesn't require calculating gate sizes

Gate Sizes

How wide should the gates be for the least delay?

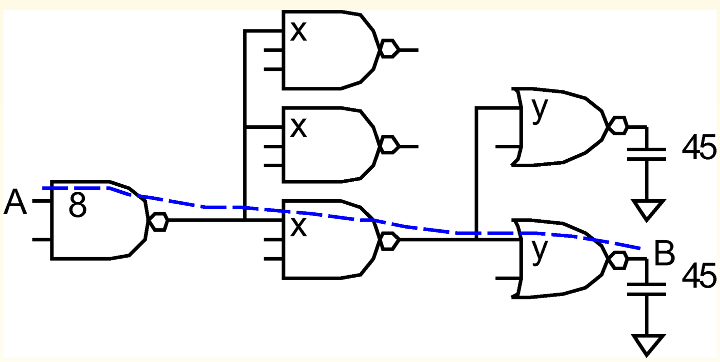
$$\hat{f} = gh = g\frac{C_{out}}{C_{in}}$$

$$\implies C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}}$$

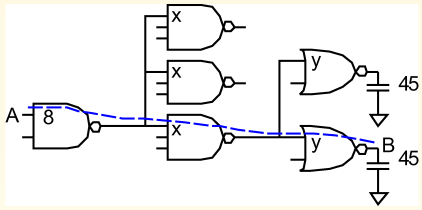
- Working backward, apply capacitance transformation to find input capacitance of each gate given load it drives
- Check work by verifying input capacitance specification is met

Example: 3-stage Path

Select gate sizes x and y for least delay from A to B



Example: 3-stage Path, Cont'd



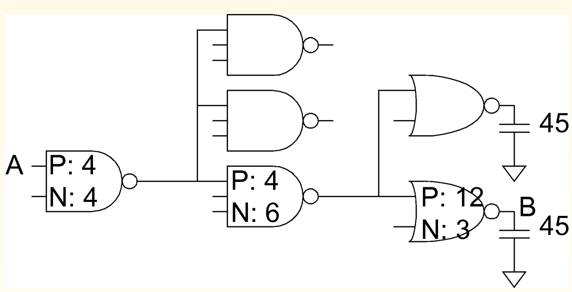
Logical Effort	G = (4/3)*(5/3)*(5/3) = 100/27
Electrical Effort	H = 45/8
Branching Effort	B = 3 * 2 = 6
Path Effort	F = GBH = 125
Best Stage Effort	$\hat{f} = \sqrt[3]{F} = 5$
Parasitic Delay	P = 2 + 3 + 2 = 7
Delay	D = 3*5 + 7 = 22 = 4.4 FO4

Example: 3-stage Path, Cont'd

Work backward for sizes

y = 45 * (5/3) / 5 = 15

x = (15*2) * (5/3) / 5 = 10



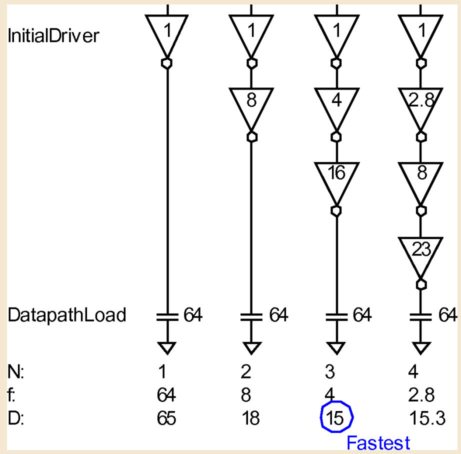
Best Number of Stages

How many stages should a path use?

- Minimizing number of stages is not always fastest
- Example: drive 64-bit datapath with unit inverter

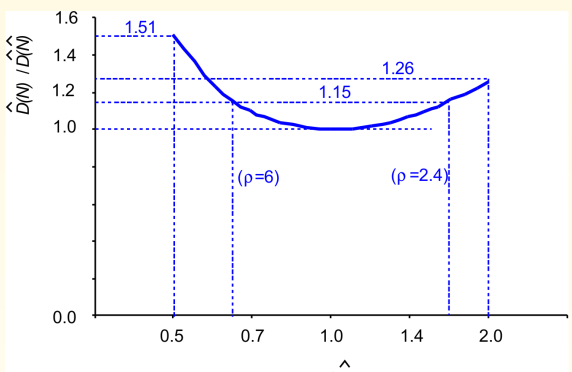
$$D = NF^{\frac{1}{N}} + P$$

$$D = N(64)^{\frac{1}{N}} + N$$



Sensitivity Analysis

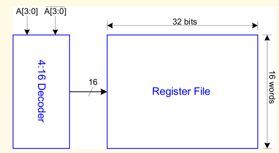
How sensitive is delay to using exactly the best number of stages?



- $2.4 < \rho < 6$ gives delay within 15% of optimal
  - We can be sloppy
  - For example, use $\rho = 4$

Decoder Example: Number of Stages

- 16 word, (32 bit) register file
- Each bit presents load of 3 unit-sized transistors
- True and complementary address inputs A[3:0]
- Each input may drive 10 unit-sized transistors



Find: number of stages, sizes of gates, speed

- Decoder effort is mainly electrical and branching
  - Electrical Effort: H = (32*3)/10 = 9.6
  - Branching Effort: B = 8
- If we neglect logical effort (assume G = 1)
  - Path Effort: F = GBH = 76.8
- Number of Stages: N = $\log_4$F = 3.1
- Try a 3-stage design

Decoder: Gate Sizes and Delay

Logical Effort: G = 1 * 6/3 * 1 = 2

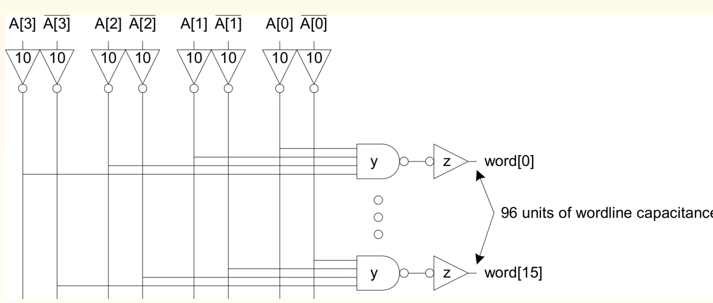
Path Effort: F = GBH = 154

Stage Effort: $\hat{f} = F^{\frac{1}{3}}$ = 5.36

Path Delay: $D = 3\hat{f} + 1 + 4 + 1$ = 22.1

Gate sizes: z = 96*1/5.36 = 18

Gate sizes: y = 18*2/5.36 = 6.7



Decoder: Comparison

Compare many alternatives with a spreadsheet

Design	N	G	P	D
NAND4-INV	2	2	5	29.8
NAND2-NOR2	2	20/9	4	30.1
INV-NAND4-INV	3	2	6	22.1
NAND4-INV-INV-INV	4	2	7	21.1
NAND2-NOR2-INV-INV	4	20/9	6	20.5
NAND2-INV-NAND2-INV	4	16/9	6	19.7
INV-NAND2-INV-NAND2-INV	5	16/9	7	20.4
NAND2-INV-NAND2-INV-INV-INV	6	16/9	8	21.6

Review of Definitions

Term	Stage	Path
Number of stages	1	N
Logical effort	g	$G = \prod g_i$
Electrical effort	$h = \frac{C_{out}}{C_{in}}$	$H = \frac{C_{out-path}}{C_{in-path}}$
Branching effort	$b = \frac{C_{on-path}+C_{off-path}}{C_{on-path}}$	$B = \prod b_i$
Effort	f = gh	F = GBH
Effort delay	f	$D_F = \sum f_i$
Parasitic delay	p	$P = \sum p_i$
Delay	d = f + p	$D = \sum d_i = D_F + P$

Method of Logical Effort

1. Compute path effort $F = GBH$
2. Estimate best number of stages $N = \log_4 F$
3. Sketch path with N stages
4. Estimate least delay $D = NF^{\frac{1}{N}} + P$
5. Determine best stage effort $\hat{f} = F^{\frac{1}{N}}$
6. Find gate sizes $C_{in} = \frac{g_i C_{out}}{f}$

Limits of logical effort

- Chicken and egg problem
  - Need path to compute G
  - But, don't know number of stages without G
- Simplistic delay model, neglects input rise time effects
- Interconnect
  - Iteration required in designs with significant wires
- Maximum speed only
  - Not minimum area/power for constrained delay

Summary of Logical Effort

Logical effort is useful for thinking of delay in circuits

- Numeric logical effort characterizes gates
- NANDs are faster than NORs in CMOS
- Paths are fastest when effort delays are $\sim$4
- Path delay is weakly sensitive to stages, sizes
- But using fewer stages doesn't mean faster paths
- Delay of path is about $\log_4$F FO4 inverter delays
- Inverters and NAND2 best for driving large caps
- Provides language for discussing fast circuits, but requires practice to master

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