Enabling Efficient Analog Synthesis by Coupling Sparse Regression and Polynomial Optimization

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Introduction: Automated Analog Synthesis

- **Simulation-based Framework**
  - Direct SPICE simulation to drive the optimization
  - Higher accuracy of point evaluations and ease of use
  - SPICE simulation is time consuming
  - Convergence to local optimal solutions due to the lack of problem structure
- **Equation-based Framework**
  - Analytical equations to drive the optimization
  - Faster meta-models instead of SPICE simulation
  - Global structure can be captured to overcome the sub-optimality

Previous Work

- **Geometric Programming (GP)**
  - Efficient solution using convex optimization
  - compatible with most analog performance functions
  - First-principle equations are not accurate for nano-scale technology
  - General posynomial fitting is NP-hard
- **Polynomial Optimization (POP)**
  - Increase model accuracy by permitting non-linearity and non-convexity
  - SDP relaxation guarantees theoretical convergence to global optimum
  - Computational intractable for high-order polynomials

Contribution: Coupling Fitting with Optimization

- **Exploiting structured sparsity to increase accuracy while maintaining tractable problem size**
- **Novel formulation of regression through overlapping group-lasso**
- **Introduction of a greedy OMP-based algorithm for efficient implementation**

General Polynomial Optimization and SDP relaxation

- **Polynomial optimization problems Formulation:**

$$\begin{aligned}\text{minimize}_x :\ & f_0(x)\\ \text{subject to} :\ & f_i(x) \geq 0, \quad i = 1, \dots, p.\end{aligned}$$

- **Semidefinite Programming (SDP) relaxation**

$$\begin{aligned}\text{minimize}_{\{m_\alpha\}} :\ & \sum_\alpha p_\alpha m_\alpha\\ \text{subject to} :\ & M_r(m) \succeq 0,\\ & M_{r-r_i}(f_i m) \succeq 0.\end{aligned}$$

- **Computational complexity characterized by dimension n, relaxation order r as $O(n^r)$**

Exploiting Sparsity in Polynomial Optimization

- **Variable Decompostion of Input Set**
  Decompose $I = \{x_1, \dots, x_n\}$ into smaller subsets $I_k \subseteq I$
  - Running Intersection Property
    $\forall k \leq p-1$, there must exist an s such that

$$I_{k+1} \cap \bigcup_{j=1}^{k} I_j \subseteq I_s.$$

  - SDP relaxation for sparse polynomial optimization

$$\begin{aligned}\text{minimize}_{\{m_\alpha\}} :\ & \sum_\alpha p_\alpha m_\alpha\\ \text{subject to} :\ & M_r(m, I_k) \succeq 0, \quad k = 1, \dots, p\\ & M_{r-r_i}(f_i m, I_k) \succeq 0, \quad k = 1, \dots, p,\end{aligned}$$

  - Variable decomposition help to reduce SDP complexity to $O(p \times max(card(I_k))^r)$
- **Correlative-sparsity-pattern (csp) matrix**
  - Optimal variable decomposition is NP-hard.
  - csp matrix as an alternative measure for sparsity

$$R_{ij} = \begin{cases} 1, \text{ if } i = j, \\ 1, \text{ if } \alpha_i, \alpha_j \geq 1, \exists \alpha \in \text{supp}(f_0), \\ 1, \text{ if } x_i \in x_{f_k} \text{ and } x_j \in x_{f_k}, k \geq 1, \\ 0, \text{ otherwise.} \end{cases}$$

Sparse Polynomial Fitting

- **Intuition about physical nature**
  - Strong coupling exists among small subsets of transistors
- **Objective**
  - Accurate polynomials with structured sparsity for optimization
  - Minimize the cardinality of csp matrix

Regression as Group Lasso

- **Group Monomials according to contributions of csp matrix**

$$\begin{aligned}G_0 &= \{\alpha\}, card(supp(a)) \geq 1\\ G_{ij} &= \{\alpha\}, \alpha_i \alpha_j \geq 1, i < j.\end{aligned}$$

- **ℓ_1/ℓ_2 regularized formulation**

$$\Omega^G_{overlap}(\omega) = \inf_{v \in V_G, \sum_{g \in G} v_g = \omega} \sum_{g \in G} \|v_g\|,$$

$$\min_{C \in \mathbb{R}^l} \frac{1}{2} \|Y - \tilde{X} C\|^2 + \lambda \Omega^G_{overlapping}(C).$$

- **OMP-based greedy algorithm**
  - Reverse grouping from csp-matrix to polynomial templates

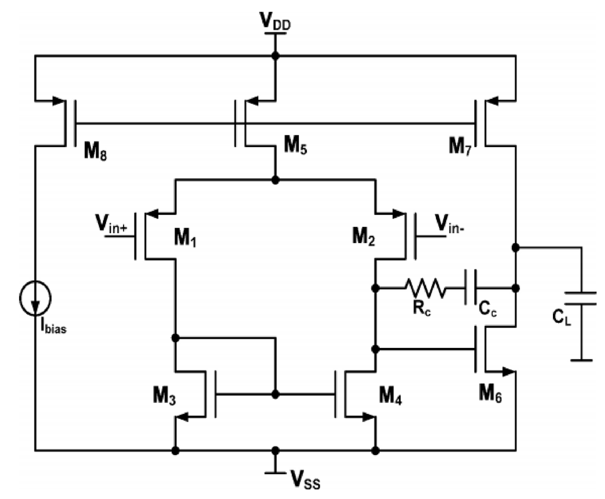
$$M(R, d) = \{\alpha, \sum_i \alpha_i \leq d, \alpha_i \alpha_j = 0 \text{ if } R(i,j) = 0\}.$$

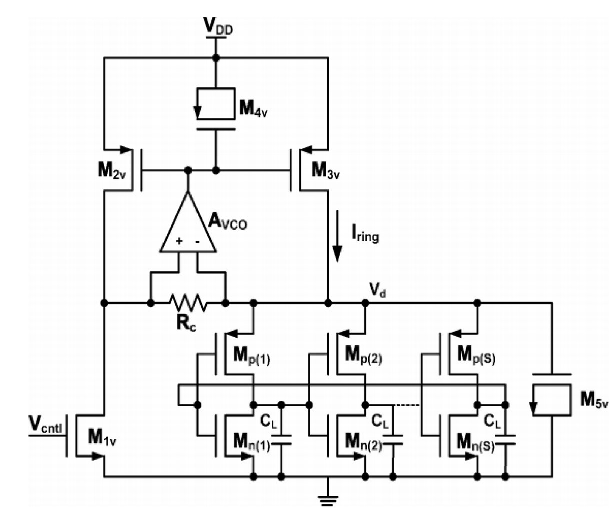
  - Seek the best matching group in csp-matrix at each iteration

Dealing with Constrained Optimization

- **More stringent sparsity for constraints**
  - $R(i,j) = 1$ if any f_k involves x_i and x_k
- **Binary search for optimal lagrangian duals**
  - Increase the multiplicative weights whose constraints are not satisfied

Results: OPAMP & VCO





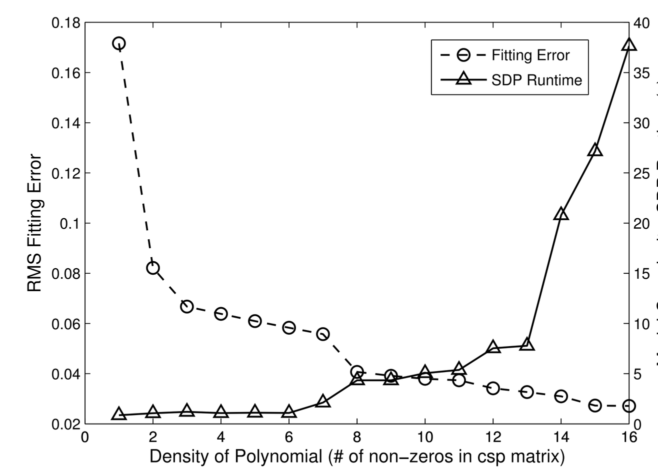
- OPAMP in Experiments VCO in Experiments

	Training Error		Test Error	
Method	Max Error	RMS Error	Max Error	RMS Error
Monomial	581%	36.5%	458%	36.4%
Quad Poly	145%	9.17%	58.1%	9.24%
C-SFO	61.3%	3.73%	34.2%	3.69%

- Model Error Reduction for OPAMP Gain

			C-SFO		Quad Poly		GP	
Case #	Metric	Spec	Model	SPICE	Model	SPICE	Model	SPICE
Case 1	Gain (10^4)	max	1.69	1.68	1.74	1.68	1.95	0.78
	UGB (MHz)	≥10	10.2	10.5	9.98	9.96	10	9.76
	PM (°)	≥60	60.6	60.5	60	60	60	58.2
Case 2	Gain (10^4)	≥1.5	1.55	1.53	1.5	1.36	1.5	0.48
	UGB (MHz)	max	12.6	12.6	14.8	14.8	18.4	17.8
	PM (°)	≥60	61.3	60.0	60	60.7	60	59.71
Case 3	Gain (10^4)	≥1.50	1.54	1.52	1.49	1.37	1.5	0.57
	UGB (MHz)	max	18.9	19.0	14.9	14.8	25.6	25.6
	PM (°)	≥48	47.8	46	48	47.95	48	48.1

- Optimization Results for OPAMP



Threshold	C-SFO	Quad Poly
0.1%	42.5%	0%
1%	77.9%	0%
2.5%	80.3%	1.6%
3%	81.1%	22.0%
4%	85.8%	73.2%
5%	88.9%	79.5%

- Sparsity & Complexity Trade-off Constraint Satisfaction

	Training Error		Test Error	
Method	Max Error	RMS Error	Max Error	RMS Error
Monomial	15.9%	2.6%	9%	2.7%
Quad Poly	4.1%	1.03%	5.8%	1.5%
C-SFO	1.5%	0.17%	0.68%	0.15%

- Error Reduction for VCO Max Frequency

			C-SFO		Quad Poly		GP	
Case #	Metric	Spec	Model	SPICE	Model	SPICE	Model	SPICE
Case 1	Power	min	4.17	4.17	3.30	4.07	N/A	N/A
	FMax (MHz)	≥2.5	2.507	2.507	2.5	2.43	N/A	N/A
	FMin (MHz)	≤0.5	0.479	0.479	0.423	0.46	N/A	N/A
Case 2	Power	min	3.054	3.054	2.9	3.45	3.11	3.02
	FMax (MHz)	≥2	2.00	2.00	2.00	1.98	2.00	1.97
	FMin (MHz)	≤0.5	0.369	0.36	0.36	0.40	0.39	0.36
Case 1	Power	min	5.44	5.43	4.74	5.41	N/A	N/A
	FMax (MHz)	≥3	3.01	3.021	3.00	2.97	N/A	N/A
	FMin (MHz)	≤0.6	0.598	0.599	0.59	0.595	N/A	N/A

- Optimization Results for VCO