# Novel Power Grid Reduction Based on L1 Regularization

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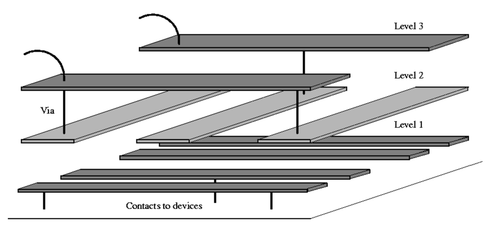
## Power Grid Reduction: Motivation

- Large-scale power grids



- Simulation time determined by size of power grid: number of ports and elements (R,L,C)
- Speed up simulation/analysis by reducing grid size
  - Focus only on steady-state analysis (DC), hope to extend to other types of analysis (Transient, AC)

## Power Grid Reduction: Formulation

- Port relation (Ohm's Law) by $L_G$

$$L_G v = i$$

  - $v$: voltages at each port
  - $i$: net currents injected into each port
- Admittance matrix (Graph Laplacian)

$$L_G(i,j) = \begin{cases} \sum_{k,k\neq i}\omega_{ik}, & \text{if } i = j, \\ -\omega_{ij}, & \text{if } i \neq j \text{ and } \{i,j\} \in E, \\ 0, & \text{otherwise.} \end{cases}$$

- Goal: want a sparse approximation $L_{G'}$
  - With far fewer non-zeros
  - Preserve similar port relation

## State-of-the-art Methods

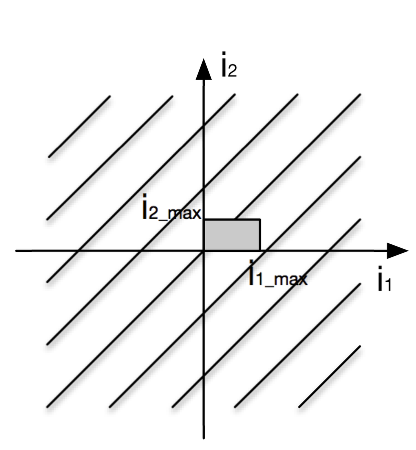
- Krylov subspace based methods [FL04,LS06]
  - Project the original system onto a low-rank Krylov subspace for efficiency
- Time-Constant Equilibration Reduction (TICER) [She99]
  - Eliminate low-degree nodes by connecting their neighbors
- Algebraic multigrid methods [SAN03]
  - Reduce the number of nodes and edges simultaneously
- Sampling based spectral sparsification approach [ZFZ14]
  - In time $O(m \log n/\epsilon^2)$, find an $\epsilon$-power approximation $G'$ of $O(n \log n/\epsilon^2)$ edges satisfies:

$$(1-\epsilon)v^T L_G v \leq v^T L_{G'} v \leq (1+\epsilon) v^T L_G v, \ \forall v \in \mathbb{R}^n.$$

- **They all try to build sparsifiers preserve $L_G v = i$ for all $i \in \mathbb{R}^n$... Is that necessary?**

## Our Key Observation

- In practice, currents delivered from ports do not vary unboundedly
  - Peak values of currents can be estimated from system-level description or transistor-level simulation
- The actual space is a small subset of the entire space




- **How to utilize the range information not explored before?**
  - For more sparsity and accuracy of reduced power grids

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## Our Main Contribution

- Propose an efficient method that **allows using range information** for better sparsification
- Leverage recent advances of $\ell^1$ regularization to drive sparsity
- We call it graph Sparsification by $\ell^1$ regularization on Laplacian (**SparseLL**)

## First Attempt for Sparsification

- Objective function: averaged error in the given range

$$\min_{L_{G'}} \int_{\Omega_V} [\|(L_G - L_{G'})v\|_2^2] dv$$

  - Allow to incorporate the range information
- Constraints: sparsity specified by $\ell^0$-norm (number of non-zeros)

$$\|L_{G'}\|_0 \leq m_0$$

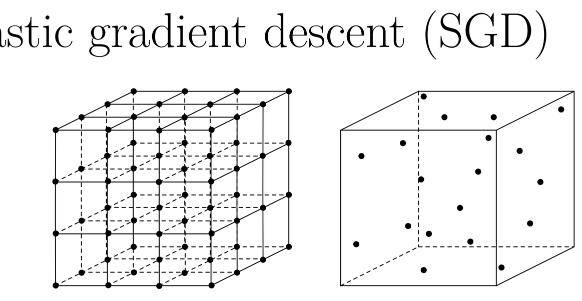
- Non-linear and non-convex in both objective and constraints, hard to solve...

## Reformulation as Stochastic Optimization

- Integral discretization by deterministic mesh requires exponential number of samples

$$\int_{\Omega_V} [\|(L_G - L_{G'})v\|_2^2] dv \approx \frac{1}{N}\sum_{i=1}^{N} \|(L_G - L_{G'})v_i\|_2^2$$

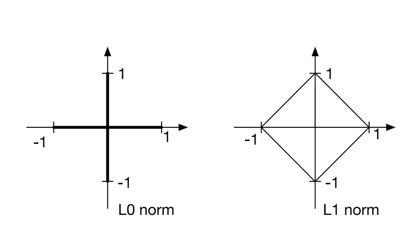
- Randomized discretization leverages fast convergence from stochastic gradient descent (SGD)



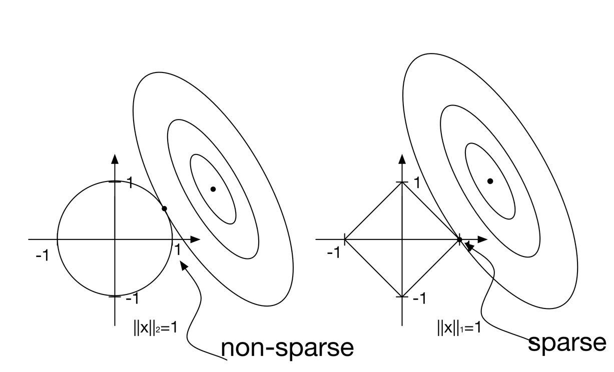

  - Sample $v_i \sim \Omega_V$, calculate gradient, update solution
  - Converge to optimal solution with rate $O(\sqrt{1/t})$ for $t$ iterations

## $\ell^1$ Regularization for Sparsity

- $\ell^0$ constraints are combinatorial and non-convex: result in an NP-hard problem
- $\ell^1$ norm is the tightest while being convex relaxation of $\ell^0$ norm




- Sparsity encouraged by spiky $\ell^1$ norm




## Complete SparseLL Formulation

- Objective: regularized empirical risk function

$$\min_{L_{G'}} \frac{1}{N}\sum_{i=1}^{N} \|(L_G - L_{G'})v_i\|_2^2 + \lambda \|L'_G\|_1$$

  - Parameter $\lambda$ controls the degree of sparsity
- Constraints:

$$L_{G'}(i,j) \leq 0, \text{ with } i \neq j,$$
$$L_{G'} = L_{G'}^T,$$
$$\sum_{j=1}^{n} L_{G'}(i,j) = 0, \forall i \in \{1,2,\ldots,n\}.$$

## Efficient Stochastic Gradient Descent Algorithm

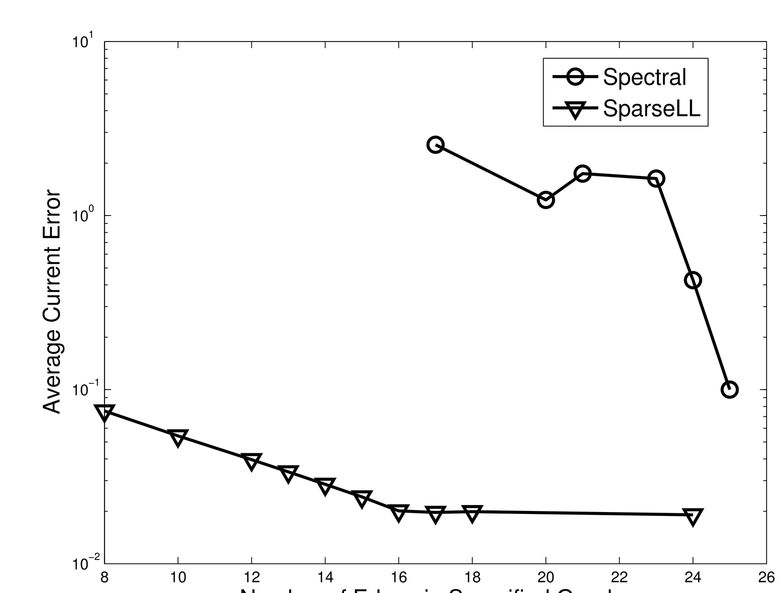
- Pseudocode

**procedure** SparseLL($L_G$, $D$, $\lambda$, $\epsilon$)
initialization
  $t = 0$
  $L_0 = 0$
  **while** accuracy above $\epsilon$ **do**
    $t = t + 1$;
    Sample a current $i$ randomly from $\Omega_I$ by $\beta \sim D$;
    Get the corresponding $v$ by solving $L_G v = i$;
    Calculate the gradient $Grad = \nabla_{L_t}(\frac{1}{2}\|(L_G - L_t)v\|_2^2) + \lambda I$
    Update graph Laplacian by $L_{t+1} = L_t - \gamma_t Grad$;
    Project $L_{t+1}$ in order to be a valid Laplacian;
  **end while**
**end procedure**

- Time complexity $O(n^2 \text{polylog}(n)/\epsilon^2)$

## Experimental Results

- Optimizing a sample 17-node power grid




- A smaller error while significantly reducing # of edges

| | | | Spectral [ZFZ14] | | SparseLL | | |
|---|---|---|---|---|---|---|---|
| | Nodes | Edges | Error | Edges | Error | Edges | Err Reduction |
| rand1 | 100 | 4000 | 5.40% | 1031 | 0.18% | 996 | 30X |
| rand2 | 500 | 100000 | 4.44% | 8120 | 0.07% | 8021 | 60X |
| rand3 | 1000 | 400000 | 4.80% | 15114 | 0.03% | 14213 | 160X |
| ibmpg1 | 5388 | 27000 | 3.80% | 6703 | 0.01% | 6570 | 380X |

- Runtime comparison

| Benchmark | SparseLL | Spectral |
|---|---|---|
| ibmpg0 | <1s | <0.01s |
| rand1 | 1s | 0.02s |
| rand2 | 5s | 0.5s |
| rand3 | 20s | 3.1s |
| ibmpg1 | 140s | 3.2s |

## References

[Bot10] Léon Bottou, *Large-scale machine learning with stochastic gradient descent*, COMPSTAT, 2010.

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[ZFZ14] Xueqian Zhao, Zhuo Feng, and Cheng Zhuo, *An efficient spectral graph sparsification approach to scalable reduction of large flip-chip power grids*, ICCAD, 2014.

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